CS350 — Winter 2019
Homework 0

Due Tuesday 15^{th} January, on paper (handwritten/typed), at the start of class. This assignment will be graded. The material covered by this homework should be familiar to you from the prerequisite courses. If anything here seems new or difficult, take this as a sign that you may not be prepared for this course. What to do? Review your notes from CS 250 and 251, read about induction and logarithms, form a study group to talk about these things with your fellow students, visit the Teaching Assistant or the Professor.

When you are doing proofs it is important to clearly mention what you are trying to prove, your assumptions and the properties you are applying to the equation. More notes on what I expect to see when you present a proof are at
http://web.cecs.pdx.edu/~black/cs350/Lectures/example%20proof.pdf

1. Read the descriptions of the three algorithms for computing GCD that appear in Levitin §1.1: Euclid's Algorithm, the Consecutive Integer checking algorithm, and the middle-school procedure.

   Implement Euclid's algorithm and the consecutive integer checking algorithm in your favorite programming language. Measure their execution times, and compare how long each takes to run on pairs of natural numbers of 8, 16, 32, 64, 128, 256 and 512 digits. (Depending on your language, you may need to use a special package to represent large integers.)

   What you should turn in: a table of results.

   Hint: Since I'm not specifying language, computer, numeric package, or the way that you measure your timings, you will need to give me that information for your answers to be reproducible.

2. **Use induction** to prove that
$$2^{n+1} - 1 = 2^0 + 2^1 + 2^2 + \cdots + 2^n$$

3. Prove, from the definition of logarithm (without using the base-change rule), that
$$a^{\log_b x} = x^{\log_b a}$$

4. Prove that
$$\log_x ab = \log_x a + \log_x b$$

5. **Use induction** to prove that
$$2^n < n! \text{ is true } \forall\ \mathbb{N} \text{ where } n \geqslant 4$$
   Be sure to *state* your induction hypothesis, and to *use* your induction hypothesis in your proof.