

CS350 – Winter 2019

Homework 1

Due Thursday, 24th January, on paper, at the start of class. This assignment will be graded.

1. For each of the following pairs of functions $f(n)$ and $g(n)$, state whether $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$, $f(n) \in \Theta(g(n))$, or none of the above. Briefly explain your reasoning.

(a) $f(n) = 2n^3 + 3n + 4$,	$g(n) = 57n + 75$
(b) $f(n) = \lg(2n + 1)$,	$g(n) = 23\sqrt{n}$
(c) $f(n) = \frac{n^3 + n}{3^{-2}}$,	$g(n) = n^2(2n + 1)$
(d) $f(n) = \frac{2^n - n^2}{100} + 3^n$,	$g(n) = 5n^4 + 3n^2 + 7$

2. What is the worst-case running time of the following function? Use big- O notation and show your work.

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pesky(n):
  r ← 0
  for i ← 1 to n do:
    for j ← 1 to i do:
      for k ← j to (i + j) do:
        r ← r + 1

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3. Consider the following eighteen functions:

$\sqrt[3]{n}$	n	2^n
$n \log n$	$n - n^3 + 7n^5$	$n^2 + (\log n)^2$
n^2	n^3	$\log n$
$n^{1/3} + \log n$	$(\log n)^2$	$n!$
$\ln n$	$\frac{n}{\log n}$	$\log \log n$
$(\frac{1}{3})^n$	$(\frac{3}{2})^n$	$1/\log n$

List these functions in increasing rate of growth. Group any two functions f and g if and only if f and g have the same rate of growth, that is, iff $f(n) \in O(g(n))$ and $g(n) \in O(f(n))$.

4. Solve the following recurrences exactly, using backward substitution.

- (a) $T(1) = 8$, and for all $n \geq 2$, $T(n) = 2T(n - 1) - 5$
 (b) $T(1) = 3$, and for all $n \geq 2$, $T(n) = T(n - 1) + n - 1$
 (c) $T(1) = 1$, and for all $n \geq 2$ such that n is a power of 2, $T(n) = 2T(\frac{n}{2}) + 2n$
 (d) $T(1) = 1$, and for all $n \geq 2$ such that n is a power of 3, $T(n) = T(\frac{n}{3}) + n$