# CS 350 Algorithms and Complexity

*Winter 2019*

## Lecture 5: Brute Force Algorithms

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# What is Brute Force?

- force of the computer, not of your intellect

  = simple & stupid

  just do it!

# Why study them?

- Simple to implement

  suppose you need to solve only one instance?
- Often “good enough”, especially when *n* is small
- Widely applicable
- Actually OK for some problems, e.g., Matrix Multiplication
- Can be the starting point for an improved algorithm
- “Baseline” against which we can compare better algorithms
- Can be a “gold standard” of correctness

  use as oracle in unit tests

# Sequential Search

```
searchFor: needle
    "sequential search for needle.  Returns true if found."

    self do: [ :each |
        (each == needle) ifTrue: [ ↑ true ].
    ].
    ↑ false
```

# Sequential Search

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searchFor: needle
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    self do: [ :each |
        (each == needle) ifTrue: [ ↑ true ].
    ].
    ↑ false
```

```
searchUsingSentinal: needle
    "sequential search for needle.  Returns true if found."

    | i |
    i ← 1.
    [(self at: i) == needle ] whileFalse: [ i ← i + 1 ].
    ↑ (i ~= self size)
```

# Sequential Search

```
searchFor: needle
    "sequential search for needle.  Returns true if found."

    self do: [ :each |
        (each == needle) ifTrue: [ ↑ true ].
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    ↑ false
```

```
searchUsingAt: needle
    "sequential search for needle.  Returns true if found."

    | i sz |
    sz ← self size.
    i ← 1.
    [((self at: i) == needle) | (i = sz) ] whileFalse: [ i ← i + 1 ].
    ↑ (i ~= sz)
```

# Sequential Search

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searchUsingAt: needle
    "sequential search for needle.  Returns true if found."

    | i sz |
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    i ← 1.
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    ↑ (i ~= self size)
```

# Timing Sequential Search

**testSequentialSearch**

```
| A B  N M res t1 t2 t3|
N ← 100000.
M ← 5000000.    " bigger than the array to be searched, and any value in it"
 A ← self randomArrayOfSize: N.
t1 ← Time millisecondsToRun: [1000 timesRepeat:   [res ← A searchFor: M ]].
self deny: res.
B ← A copyWith: M.
t2 ← Time millisecondsToRun: [1000 timesRepeat:  [res ← B searchUsingSentinel: M ]].
self deny: res.
t3 ← Time millisecondsToRun: [1000 timesRepeat:  [res ← A searchUsingAt: M ]].
self deny: res.
Transcript show: 'Sequential search, size: '; show: N; cr;
        show: ' sequential, for each: '; show: t1; show: 'µs'; cr;
        show: ' with sentinel: '; show: t2; show: 'µs'; cr;
        show: ' without sentinel, at: '; show: t3; show: 'µs'; cr; cr.
```

# Timing Results

Sequential search, size: 100000
sequential, for each: 1430µs
with sentinel: 850µs
without sentinel, at: 1287µs

Sequential search, size: 100000
sequential, for each: 1396µs
with sentinel: 788µs
without sentinel, at: 1280µs

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Coding details *can* make a difference!

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Coding details ***can*** make a difference!

But ***not*** to the asymptotic complexity.

# Selection Sort

**ALGORITHM** *SelectionSort*$(A[0..n-1])$

//Sorts a given array by selection sort
//Input: An array $A[0..n-1]$ of orderable elements
//Output: Array $A[0..n-1]$ sorted in ascending order
**for** $i \leftarrow 0$ **to** $n-2$ **do**
    $min \leftarrow i$
    **for** $j \leftarrow i+1$ **to** $n-1$ **do**
        **if** $A[j] < A[min]$   $min \leftarrow j$
    swap $A[i]$ and $A[min]$

```
selectionSort
    "Sort me using selection sort. Levitin §3.1"

    | indexOfMin n A |
    A ← self.
    n ← self size.
    1 to: n - 1 do: [ :i |
        indexOfMin ← i.
        i + 1 to: n do: [ :j |
            (A at: j) < (A at: indexOfMin) ifTrue: [
                indexOfMin ← j]].
        A swap: i with: indexOfMin ]
```

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  swap $A[i]$ and $A[min]$

# Ex 3.1, Problem 4

a. Design a brute-force algorithm for computing the value of a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

at a given point $x_0$ and determine its worst-case efficiency class.

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Assume that exponentiation is *not* built-in.

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at a given point $x_0$ and determine its worst-case efficiency class.

Assume that exponentiation is *not* built-in.

Write it down clearly, so I can project it with the document camera.

# Ex 3.1, Problem 4

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b. If the algorithm you designed is in $\Theta(n^2)$, design a linear algorithm for this problem.

# Solution to Problem 4

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$$p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

at a given point $x_0$ and determine its worst-case efficiency class.

**Algorithm** $BruteForcePolynomialEvaluation(P[0..n], x)$
//The algorithm computes the value of polynomial $P$ at a given point $x$
//by the "highest-to-lowest term" brute-force algorithm
//Input: Array $P[0..n]$ of the coefficients of a polynomial of degree $n$,
// stored from the lowest to the highest and a number $x$
//Output: The value of the polynomial at the point $x$
$p \leftarrow 0.0$
**for** $i \leftarrow n$ **downto** 0 **do**
    $power \leftarrow 1$
    **for** $j \leftarrow 1$ **to** $i$ **do**
        $power \leftarrow power * x$
    $p \leftarrow p + P[i] * power$
**return** $p$

# Solution to Problem 4

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- size of input is degree of polynomial, $n$
- number of multiplications depends only on $n$
- number of multiplications, $M(n) \in$ ?

A. $\Theta(n)$

B. $\Theta(n^2)$

C. $\Theta(n \lg n)$

D. $\Theta(n^3)$

# Ex 3.1, Problem 4

a. Design a brute-force algorithm for computing the value of a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

at a given point $x_0$ and determine its worst-case efficiency class.

b. If the algorithm you designed is in $\Theta(n^2)$, design a linear algorithm for this problem.

# Solution to Problem 4

**Algorithm** $BetterBruteForcePolynomialEvaluation(P[0..n], x)$
//The algorithm computes the value of polynomial $P$ at a given point $x$
//by the "lowest-to-highest term" algorithm
//Input: Array $P[0..n]$ of the coefficients of a polynomial of degree $n$,
//         from the lowest to the highest, and a number $x$
//Output: The value of the polynomial at the point $x$
$p \leftarrow P[0]; \quad power \leftarrow 1$
**for** $i \leftarrow 1$ **to** $n$ **do**
    $power \leftarrow power * x$
    $p \leftarrow p + P[i] * power$
**return** $p$

# True or False?

- It is possible to design an algorithm with better-than-linear efficiency to calculate the value of a polynomial.

A. True

B. False

# Ex 3.1, Problem 9

- Is selection sort stable?
  - The definition of a stable sort was given in Levitin §1.3

A. Yes, it is stable

B. No, it is not stable

# Ex 3.1, Problem 10

- Is it possible to implement selection sort for a <u>linked-list</u> with the same $\Theta(n^2)$ efficiency as for an <u>array</u>?

A. Yes, it is possible

B. No, it is not possible

# BubbleSort

**ALGORITHM** $BubbleSort(A[0..n-1])$

//Sorts a given array by bubble sort
//Input: An array $A[0..n-1]$ of orderable elements
//Output: Array $A[0..n-1]$ sorted in ascending order
**for** $i \leftarrow 0$ **to** $n-2$ **do**
    **for** $j \leftarrow 0$ **to** $n-2-i$ **do**
        **if** $A[j+1] < A[j]$ swap $A[j]$ and $A[j+1]$

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- Is BubbleSort stable?

A: Yes, it is stable

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- Is BubbleSort stable?
- *Prove* that, if BubbleSort makes no *swaps* on a pass through the array, then the array is sorted.

# String Matching

# Applications:

- Find all occurrences of a particular word in a given text
  - Searching for text in an editor
  - …
- Compare two strings to see how similar they are to one another …
  - Code diff-ing
  - DNA sequencing
  - …
- …

# Notation

- Let A be a set of characters (the alphabet)
- The set of strings that consist of finite sequences of characters in A is written A* (the Kleene Star)
- For a string s, we'll write:
  - s[j] for the $j^{th}$ character in s
  - |s| for the length of s
  - s[i..j] for the substring of s from s[i] to s[j]
  - s[..n] for the prefix s[1..n], and s[m..] for s[m..|s|]
  - ε for the empty string (example: s[1..0] = ε)
  - st for the concatenation of s with another string t

# Simple Complexities:

Assume that string is represented by an array of consecutive characters

What's the worst case running time for <u>brute-force</u> testing to determine:

- whether s = t

# Simple Complexities:

Assume that string is represented by an array of consecutive characters

What's the worst case running time for brute-force testing to determine:

- whether s = t

A. O(1)
B. O(|s|)
C. O(|min(s, t)|)
D. O(|s|$^2$)
E. None of the above

# Simple Complexities:

Assume that string s is represented by an array of consecutive characters

- Worst case running time for computing s[i] ?

# Simple Complexities:

Assume that string s is represented by an <u>array</u> of consecutive characters

- Worst case running time for computing s[i] ?

A. $\Theta(1)$
B. $\Theta(|s|)$
C. $\Theta(|\min(|s|, i)|)$
D. $\Theta(i)$
E. None of the above

# Simple Complexities:

Assume that strings are represented by arrays of consecutive characters

- Worst case running time for computing st ?

# Simple Complexities:

Assume that strings are represented by arrays of consecutive characters

- Worst case running time for computing st ?

A. $\Theta(1)$
B. $\Theta(|s|)$
C. $\Theta(|\min(s, t)|)$
D. $\Theta(|\min(s, t)|^2)$
E. None of the above

# Simple Complexities:

Assume that string is represented by an array of consecutive characters

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# Simple Complexities:

Assume that string is represented by an array of consecutive characters

- Worst case running times for computing s[i..j]

A. $\Theta(1)$

B. $\Theta(|s[i..j]|)$

C. $\Theta(j-i)$

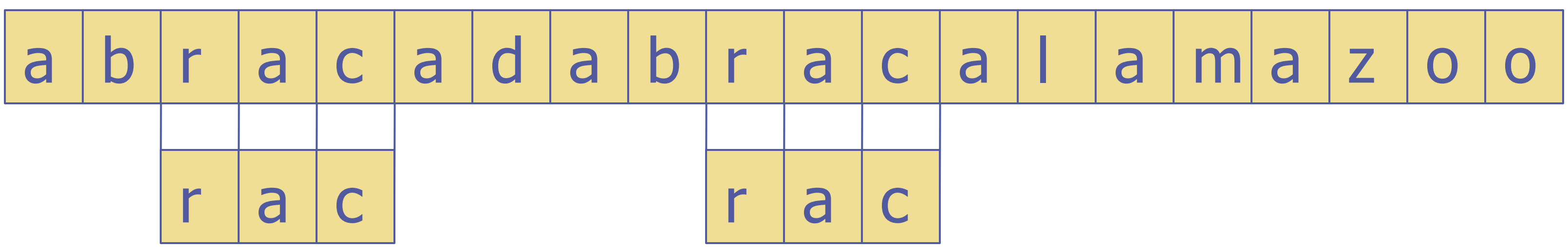
D. $\Theta((j-i)^2)$

E. None of the above

# String Matching

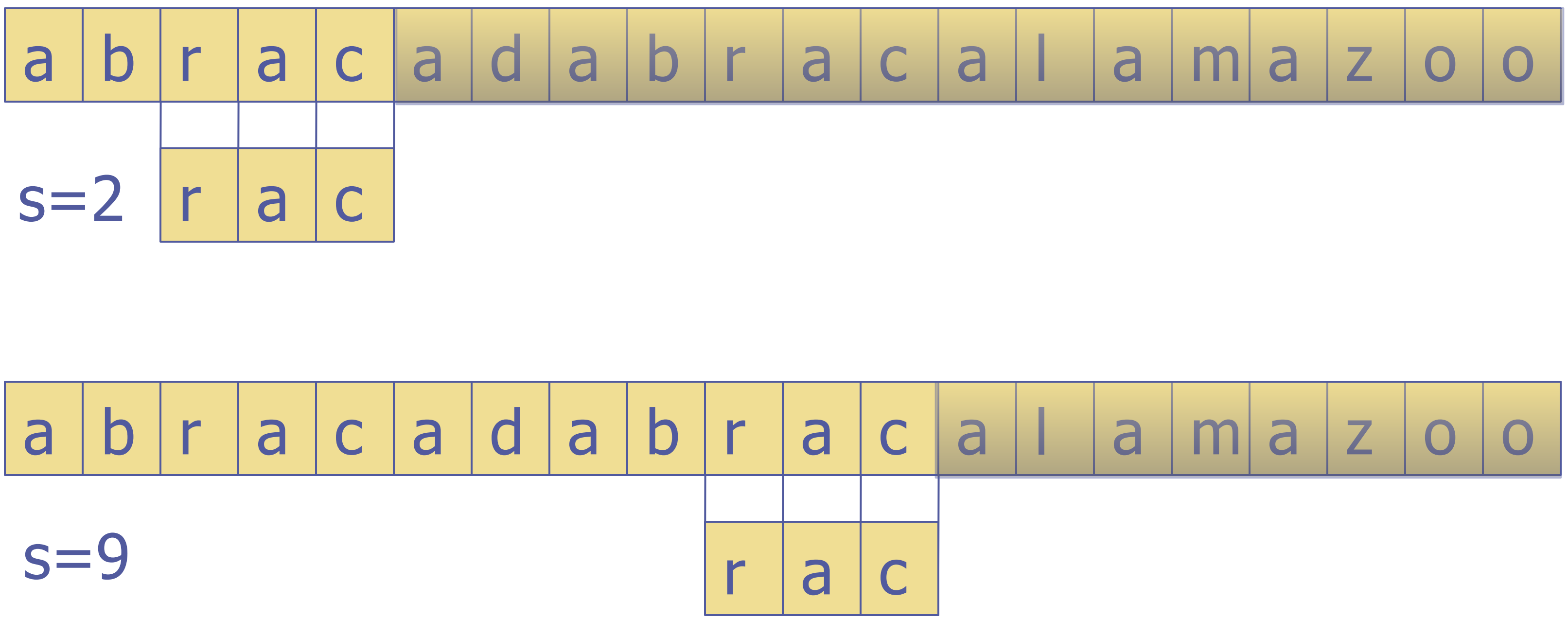
- Find <u>all occurrences</u> of a pattern string p in a text string t

- For example:



# String Matching, formally

- Given a text string, t, and a pattern string, p, of length m = |p|, find the set of all shifts s such that p = t[s+1..s+m]



# Brute-force Matching Algorithm

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |

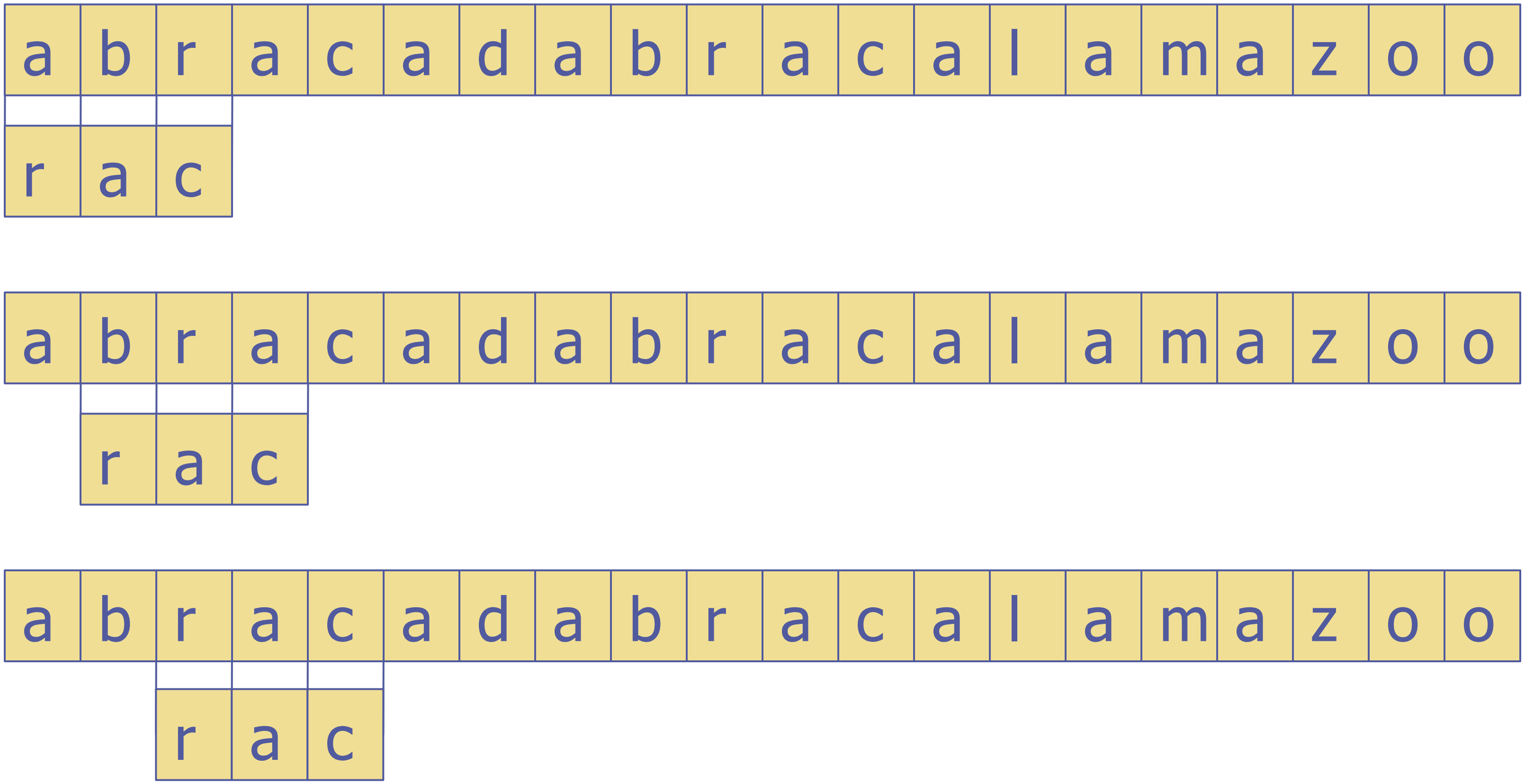
| | | |
|---|---|---|
| r | a | c |

# Brute-force Matching Algorithm

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| r | a | c | | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| | r | a | c | | | | | | | | | | | | | | | | |

# Brute-force Matching Algorithm



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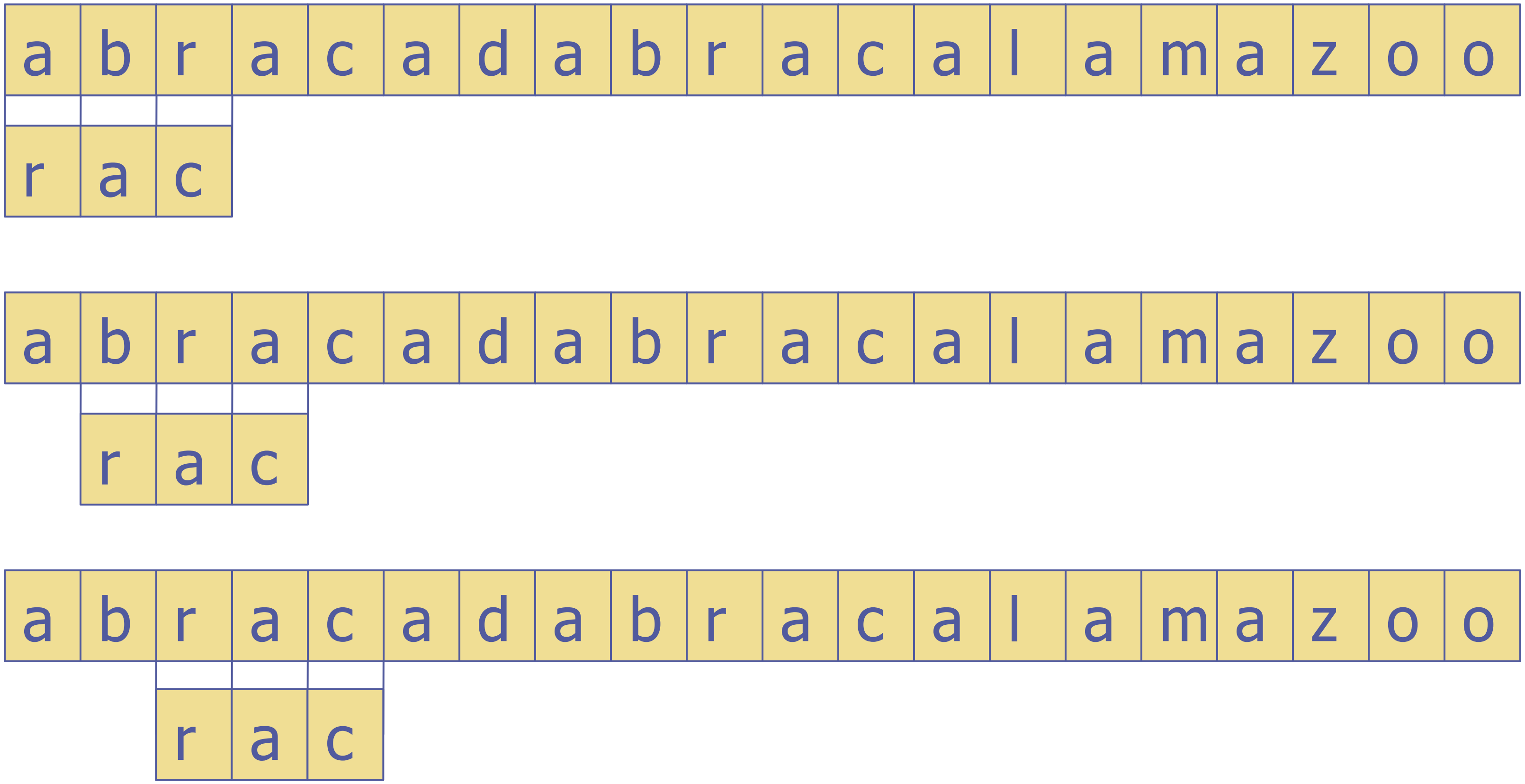


…

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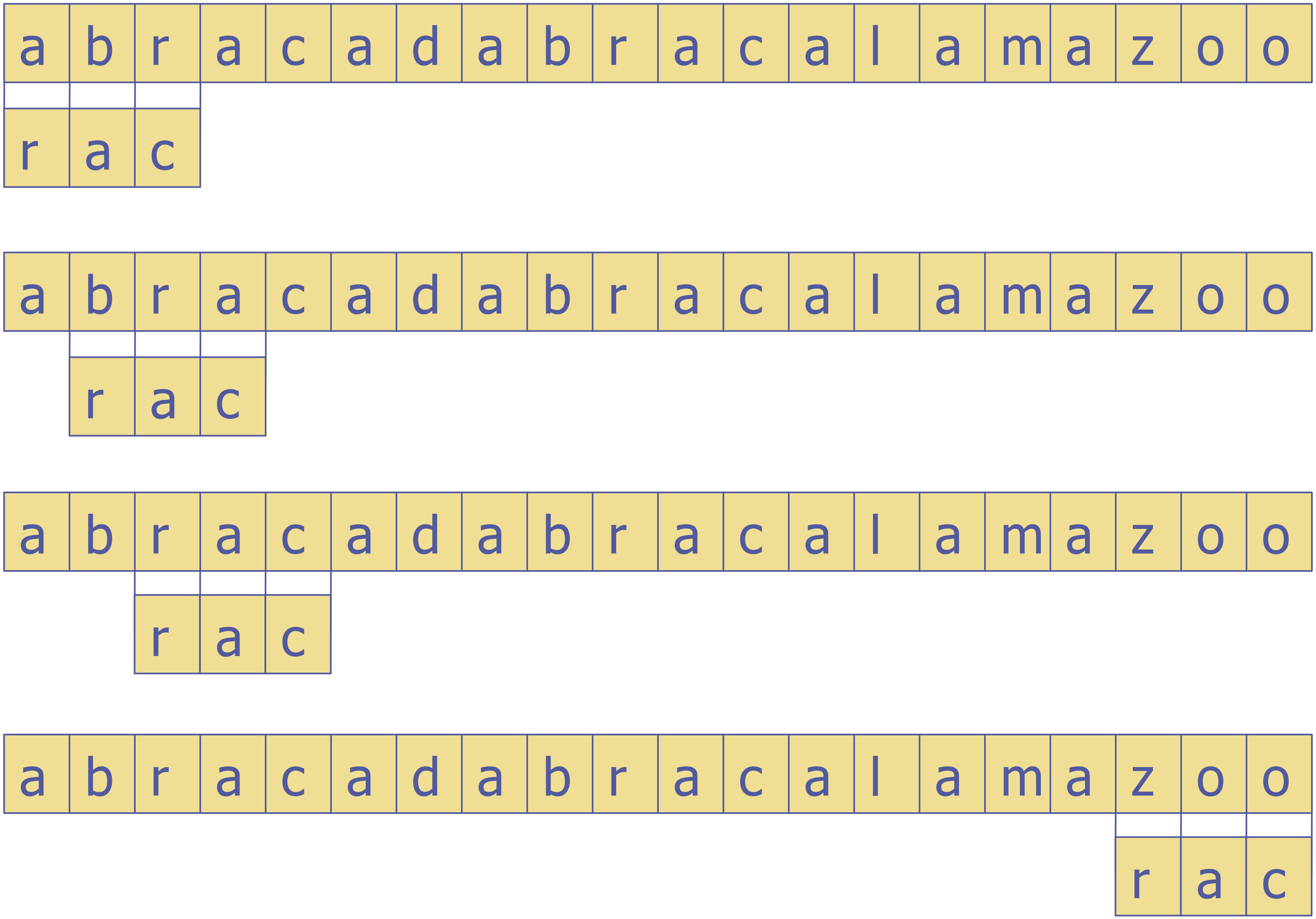

# Brute-force Matching Algorithm

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t = | a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| p = | r | a | c | | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| | r | a | c | | | | | | | | | | | | | | | | |

# Brute-force Matching Algorithm

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t = | a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| | | | | | | | | | | | | | | | | | | | | |
| p = | r | a | c | | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| | | | | | | | | | | | | | | | | | | | |
| | r | a | c | | | | | | | | | | | | | | | | |

What's the asymptotic complexity of brute-force matching?:

# Brute-force Matching Algorithm

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t = | a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| | | | | | | | | | | | | | | | | | | | | |
| p = | r | a | c | | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | r | a | c | a | d | a | b | r | a | c | a | l | a | m | a | z | o | o |
| | | | | | | | | | | | | | | | | | | | |
| | r | a | c | | | | | | | | | | | | | | | | |

What's the asymptotic complexity of brute-force matching?:

A. $\Theta(1)$
B. $\Theta(|t|)$
C. $\Theta(|p|)$
D. $\Theta(|p|(|t|-|p|+1))$
E. None of the above

# Brute-force Matching Algorithm

```
match(t, p)
  m ← |p|
  n ← |t|
  results ← {}
  for s ← 0..n-m do
    if p == t[s+1 .. s+m] then
      results ← results ∪ {s}
  return results
```

# Brute-force Matching Algorithm

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Asymptotic Complexity:
$\Theta(m(n-m+1))$

# Brute-force Matching Algorithm

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  for s ← 0..n-m do
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  return results
```

Cost of this test is O(m)

Asymptotic Complexity:
$\Theta(m(n-m+1))$

# Can we do better?

- Perhaps surprisingly: yes!
- Key insight: when a match fails, we learned something
  - Better algorithms in Chapter 7