

The Expression Problem

Gracefully

Andrew P. Black
Portland State University



The Expression Problem

Oliveira & Cook (ECOOP 2012):

*“The ‘expression problem’ (EP) [38, 10, 46] is now a classical problem in programming languages. It refers to the **difficulty** of writing data abstractions that can be easily extended with both new operations and new data variants.”*

Extensibility for the Masses Practical Extensibility with Object Algebras

Bruno C. d. S. Oliveira¹ and William R. Cook²

¹National University of Singapore
bruno@ropas.snu.ac.kr

²University of Texas, Austin
wcook@cs.utexas.edu

Abstract. This paper presents a new solution to the expression problem (EP) that works in OO languages with simple generics (including Java or C#). A key novelty of this solution is that advanced typing features, including F-bounded quantification, wildcards and variance annotations, are not needed. The solution is based on *object algebras*, which are an abstraction closely related to algebraic datatypes and Church encodings. Object algebras also have much in common with the traditional forms of the VISITOR pattern, but without many of its drawbacks: they are extensible, remove the need for accept methods, and do not compromise encapsulation. We show applications of object algebras that go beyond toy examples usually presented in solutions for the expression problem. In the paper we develop an increasingly more complex set of features for a mini-imperative language, and we discuss a real-world application of object algebras in an implementation of remote batches. We believe that object algebras bring extensibility to the masses: object algebras work in mainstream OO languages, and they significantly reduce the conceptual overhead by using only features that are used by everyday programmers.

1 Introduction

The “expression problem” (EP) [38, 10, 46] is now a classical problem in programming languages. It refers to the difficulty of writing data abstractions that can be easily extended with both new operations and new data variants. Traditionally the kinds of data abstraction found in functional languages can be extended with new operations, but adding new data variants is difficult. The traditional object-oriented approach to data abstraction facilitates adding new data variants (classes), while adding new operations is more difficult. The VISITOR Pattern [13] is often used to allow operations to be added to object-oriented data abstractions, but the common approach to visitors prevents adding new classes. Extensible visitors can be created [43, 50, 31], but so far solutions in the literature require complex and unwieldy types, or advanced programming languages.

In this paper we present a new approach to the EP based on *object algebras*. An object algebra is a class that implements a generic abstract factory interface, which corresponds to a particular kind of *algebraic signature* [18]. Object

What *is* the Expression Problem?

- Consider a simple implementation of (immutable) lists

		Operations		
		first	rest	isEmpty
Representations	ConsList (e, l)	return e	return l	false
	EmptyList	error	error	true

Algebraic data types:

- Organize program by columns

		Operations		
		first	rest	isEmpty
Representations	ConsList (e, l)	return e	return l	false
	EmptyList	error	error	true

first function

rest function

isEmpty function

Algebraic data types:

- Organize program by columns
 - easy to add a new column, but hard to add a new row

		Operations		
		first	rest	isEmpty
Representations	ConsList (e, l)	return e	return l	false
	EmptyList	error	error	true

first function

rest function

isEmpty function

Objects

- Organize program by rows

		Operations			
		first	rest	isEmpty	
Representations	ConsList (e, l)	return e	return l	false	ConsList class
	EmptyList	error	error	true	EmptyList class

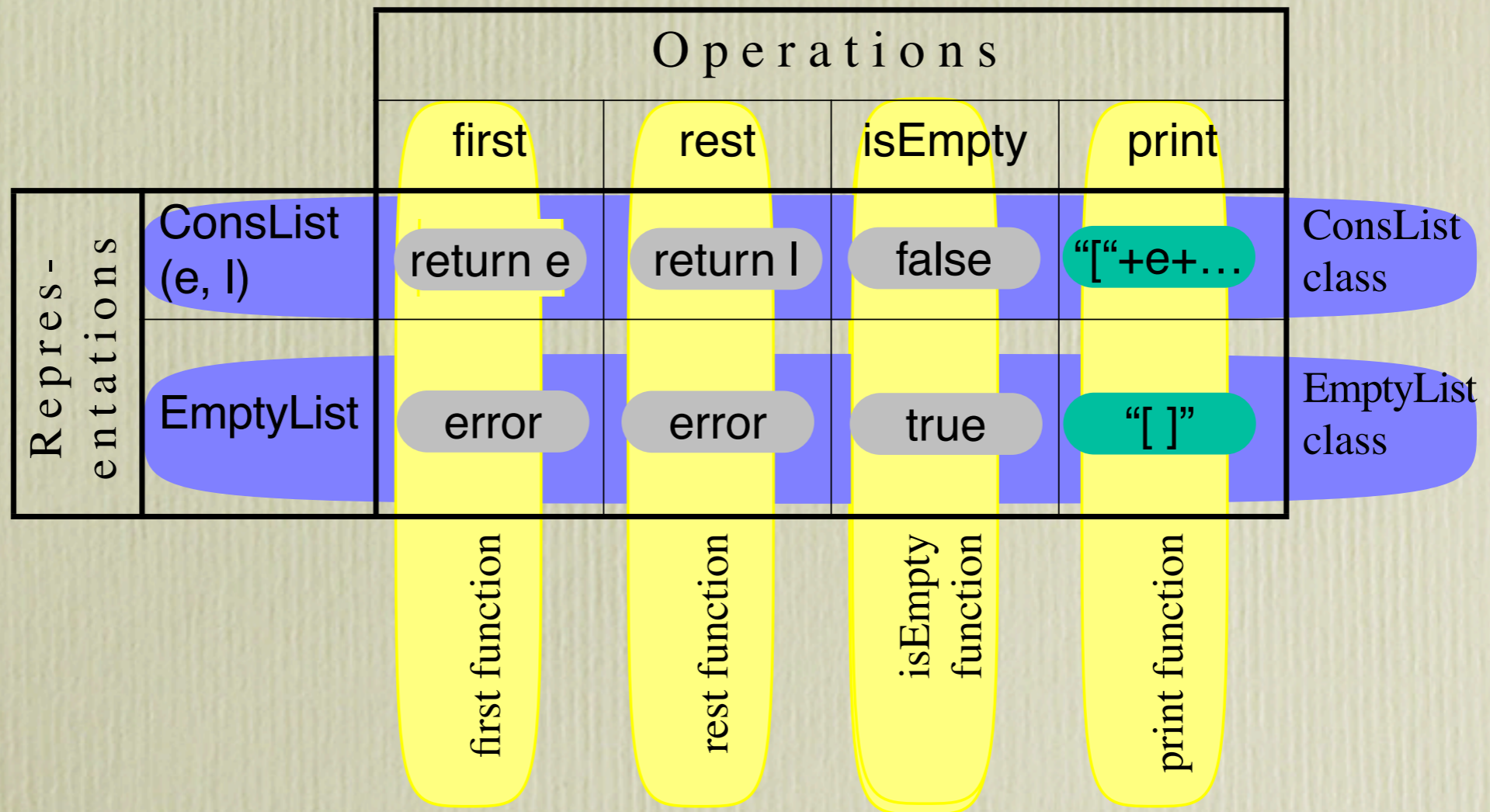
Objects

- Organize program by rows
 - easy to add a new row, but hard to add a new column

		Operations			
		first	rest	isEmpty	
Representations	ConsList (e, l)	return e	return l	false	ConsList class
	EmptyList	error	error	true	EmptyList class

Example: add an operation

- *One* new function with algebraic data, but *two* new methods in two classes with objects



Why is this “difficult”?

- An editing problem
 - assumption: adding methods to two classes involves editing two files
- A packaging problem
 - assumption: the class is the smallest unit of modularity, so editing a class breaks modularity
- A typing problem
 - assumption: fields of the objects have been given types that allow just the base operations

Some History ...

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38. Reynolds, J.C.: User-defined types and procedural data structures as complementary approaches to type abstraction. In: Schuman, S.A. (ed.) *New Directions in Algorithmic Languages*, pp. 157–168 (1975)
10. Cook, W.R.: Object-oriented programming versus abstract data types. In: *Proceedings of the REX School/Workshop on Foundations of Object-Oriented Languages*. pp. 151–178. Springer-Verlag (1991)
46. Wadler, P.: The Expression Problem. Email (Nov 1998), discussion on the Java Genericity mailing list

Some History ...

Krishnamurthi et al. captured the issue:

“A recursively defined set of data must be processed by several different tools. In anticipation of future extensions, the data specification and the tools should therefore be implemented such that it is easy to

- 1. add a new variant of data and adjust the existing tools accordingly, and*
- 2. extend the collection of tools.”*

Krishnamurthi, S., Felleisen, M., and Friedman, D. P. 1998. Synthesizing object-oriented and functional design to promote reuse. In ECOOP'98 — Object-Oriented Programming, E. Jul, Ed. LNCS vol. 1445. Springer, pp. 91–113.

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Restriction: “ideally, these extensions should not require any changes to existing code”

Some History ...

Wadler made this “problem” famous in 1998 (by coining a catchy name)

“The Expression Problem delineates a central tension in language design. Accordingly, it has been widely discussed, including Reynolds (1975), Cook (1990), and Krishnamurthi, Felleisen and Friedman (1998); the latter includes a more extensive list of references. It has also been discussed on this mailing list by Corky Cartwright and Kim Bruce. Yet I know of no widely-used language that solves The Expression Problem while satisfying the constraints of independent compilation and static typing.”

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Restrictions:

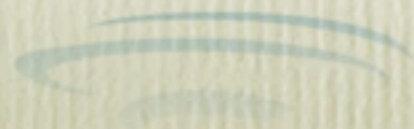
1. Static type safety (no casts)
2. No recompilation of existing code

Why wait until 1998?

- Simula '67, C++
 - have algebraic data as well as objects
- Smalltalk 80
 - classes are not the unit of modularity
- Visitor Pattern (name *Visitor* coined 1993)
 - solves the problem, at the cost of pre-planning

Why wait until 1998?

Why wait until 1998?



Why wait until 1998?



Back to the future ...

- How was this “problem” solved in Smalltalk?
 - Classes named by global *variables*
 - *methods* are the unit of compilation & packaging
 - a package contains both *new classes* (and their methods) and extensions to existing classes (*new methods*)
 - loading a package into a Smalltalk system:
 - ▶ *changes* some existing classes (overrides and adds methods, adds instance variables)
 - ▶ introduces some new classes

Snapshot Browser: Grace-Parser

*Extensions Grace-Parser	PPToken PPTokenParser PositionableStream SequenceableCollectio Stream String Text	*Grace-Parser	symbol
-----------------------------	---	---------------	--------

instance ? class

symbol
^ self asParser token trimSpaces

Monticello Browser

+Package +Slice Browse Changes Save +Repository Open History Backport

- * Grace-Parser (Grace-Parser-AndrewBlack.30) </Users/black/Development/Pharo/Pharo Grace Pars>
- * Grace-ParserTests (Grace-ParserTests-Andre <http://www.squeaksource.com/GraceParser>)
- * PetitParser (PetitParser-AndrewBlack.212)
- * System-Support (System-Support-MarcusDer)
- * Tools (Tools-MarcusDenker.407)
- AST-Core (AST-Core-lr.79)
- AST-Semantic (AST-Semantic-lr.11)
- AST-Tests-Core (AST-Tests-Core-lr.10)
- Announcements (Announcements-adrian_lienha
- ArchiveViewer (ArchiveViewer-MarianoMartinez
- AutomaticMethodCategorizer (AutomaticMetho
- AutomaticMethodCategorizerOB (AutomaticMe
- Balloon (Balloon-StephaneDucasse.49)
- BitBlitPen (BitBlitPen-StephaneDucasse.1)
- Bogus (Bogus-cwp.18)
- BogusInfo (BogusInfo-lr.22)

Snapshot Browser: Grace-Parser

*Extensions
Grace-Parser

PPToken
PPTokenParser
PositionableStream
SequenceableCollectio
Stream
String
Text

instance ? class

symbol
^ self asParser token trimSpaces

Shout Workspace

```
if symbol
```

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Snapshot Browser: Grace-Parser

*Extensions
Grace-Parser

PPToken
PPTokenParser
PositionableStream
SequenceableCollectio
Stream
String
Text

*Grace-Parser

symbol

Shout Workspace

'if' symbol

a PPTrimmingParser

▼ a PPTrimmingParser(trimSpaces)

- ▼ parser : a PPTokenParser(357826560)
 - ▼ parser : a PPLiteralSequenceParser(356253696, 'if')
 - ▶ literal : 'if'
 - ▶ message : "'if' expected'
 - ▶ properties : nil
 - ▶ size : 2
 - ▶ properties : nil
 - ▶ tokenClass : GraceToken
- ▶ properties : a Dictionary (size: 1)
- ▼ trimmer : a PPLiteralObjectParser(384565248, Character space)
 - ▶ literal : Character space
 - ▶ message : "' ' expected'
 - ▶ properties : nil

symbol
^ self asPa

Backport

aro Grace Pars
Parser

AutomaticMethodCategorizerOB (AutomaticMe
Balloon (Balloon-StephaneDucasse.49)
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▶ BogusInfo (BogusInfo-lr.22)

Why does this work?

- Classes are (mutable) objects
 - adding (or changing) a method mutates the class
- Classes are named by global *variables*
 - loading a new version of a class definition changes the value of the global variable, and recompiles all existing methods
- Objects created by a methods in a class
- No modular type-checking

Why doesn't it work in Java?

- Classes are *not* objects, and are immutable
 - Classes can be changed only by editing the source and recompiling
- Classes have global names, and cannot be renamed, assigned, or aliased
- Objects created by a language built-in **new**
- Modular type-checking

Java

```
e = new EmptyList  
o = e.append(23)
```

Smalltalk

```
e := EmptyList new  
o := e ++ 23
```


Java

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e = new EmptyList  
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Data (row) extensibility is easy: add a new package defining a new class (but also must change creation code)

Java

Smalltalk

```
e = new EmptyList
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```
e := EmptyList new
```

```
o = e.append(23)
```

```
o := e ++ 23
```

Data (row) extensibility is easy: add a new package defining a new class (but also must change creation code)

Operation (column) extensibility is impossible: can't change an existing class without editing the source.

- What about subclassing?
 - idea: subclass all of the original classes to create new variants with the additional operations.
 - Wadler focussed on generalizing the Java type system to make it possible to write those subclasses.
- But this doesn't help!
 - We still have to *change all the creation code* to use the new classes instead of the existing classes.

Grace

- new, simple O-O language
 - designed for teaching novice programmers the concepts of object-oriented programming
- block-structured within a module
- modules are objects
- no global variables
 - modules are imported under a name chosen by the *client*

Oliveira and Cook's Example

exp_base		Operations
		eval
Representations	lit(n)	n
	sum(e ₁ , e ₂)	e ₁ .eval + e ₂ .eval

Oliveira and Cook's Example

exp+pretty		Operations	
		eval	pretty
Representations	lit(n)	n	"{n}"
	sum(e ₁ , e ₂)	e ₁ .eval + e ₂ .eval	"{e ₁ .pretty} + {e ₂ .pretty}"

Oliveira and Cook's code

from their paper, translated into Grace

dialect "staticTypes"

type Value = Object

type Exp = { eval -> Value }

factory method lit(i:Number) -> Exp {

method x -> Number { i }

method eval -> Value { x }

}

factory method sum(a:Exp, b:Exp) -> Exp {

method l -> Exp { a }

method r -> Exp { b }

method eval -> Value { l.eval + r.eval }

}

// Demonstration:

def threePlusFour:Exp = sum(lit 3, lit 4)

print "{threePlusFour} = {threePlusFour.eval}"

// prints: an object = 7

exp_base.grace

Oliveira and Cook's code from their paper, translated into Grace

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exp_base.grace

```
$ apbmg exp_base.grace
self.sum[0x0x7fc6cbc1b9f8] = 7
$
```


Graceful solution

```
dialect "staticTypes"  
import "exp_base" as baseExp  
type Exp = baseExp.Exp & type { pretty -> String }
```

```
factory method lit(i:Number) -> Exp {  
  inherits baseExp.lit(i)  
  method pretty { x.asString }  
}
```

exp+pretty.grace

```
factory method sum(a:Exp, b:Exp) -> Exp {  
  inherits baseExp.sum(a, b)  
  method pretty { "{l.pretty} + {r.pretty}" }  
}
```

// Demonstration:

```
def threePlusFour:Exp = sum(lit 3, lit 4)  
print "{threePlusFour.pretty} = {threePlusFour.eval}"  
// prints: 3 + 4 = 7
```


Graceful solution

```
dialect "staticTypes"  
import "exp_base" as baseExp  
type Exp = baseExp.Exp & type { pretty -> String }
```

```
factory method lit(i:Number) -> Exp {  
  inherits baseExp.lit(i)  
  method pretty { x.asString }  
}
```

exp+pretty.grace

```
factory method sum(a:Exp, b:Exp) -> Exp {  
  inherits baseExp.sum(a, b)  
  method pretty { "{l.pretty} + {r.pretty}" }  
}
```

// Demonstration:

```
def threePlusFour:Exp = sum(lit 3, lit 4)  
print "{threePlusFour.pretty} = {threePlusFour.eval}"
```

// prints: 3 + 4 = 7

```
$ apbmg exp+pretty.grace  
self.add[0x0x7f9d40523c58] = 7  
3 + 4 = 7  
$
```


Oliveira and Cook's Example

exp and pretty		Operations	
		eval	pretty
Representations	lit(n)	n	"{n}"
	sum(e ₁ , e ₂)	e ₁ .eval + e ₂ .eval	"{e ₁ .pretty} + {e ₂ .pretty}"

Oliveira and Cook's Example

exp+pretty+bool		Operations	
		eval	pretty
Representations	lit(n)	n	"{n}"
	sum(e ₁ , e ₂)	e ₁ .eval + e ₂ .eval	"{e ₁ .pretty} + {e ₂ .pretty}"
	bool(b)	b	"{b}"
	iff(c, th, el)	if(c.eval)then {th.eval} else {el.eval}	"if {c.pretty} then {th.pretty} else {el.pretty}"

dialect "staticTypes"

import "exp_and_pretty" **as** baseExp

type Exp = baseExp.Exp

type Value = Object

method sum(l:Exp, r:Exp) -> Exp { baseExp.sum(l, r) }

method lit(x:Number) -> Exp { baseExp.lit(x) }

factory method bool(b:Boolean) -> Exp {

method x -> Boolean { b }

method eval -> Value { x }

method pretty -> String { b.asString }

}

factory method iff(c:Exp, t:Exp, f:Exp) -> Exp {

method eval -> Value {

 if (c.eval) then { t.eval } else { f.eval }

 }

method pretty -> String {

 "if ({c.pretty}) then {t.pretty} else {f.pretty}"

 }

}

def e3plus4:Exp = sum(lit 3, lit 4)

def e2plus6:Exp = sum(lit 2, lit 6)

def ett:Exp = bool(**true**)

def ifExpr:Exp = iff(ett, e3plus4, e2plus6)

print "{ifExpr.pretty} = {ifExpr.eval}"

// prints: if (true) then 3 + 4 else 2 + 6 = 7

exp+pretty+bool.grace

dialect "staticTypes"

import "exp_and_pretty" **as** baseExp

type Exp = baseExp.Exp

type Value = Object

method sum(l:Exp, r:Exp) -> Exp { baseExp.sum(l, r) }

method lit(x:Number) -> Exp { baseExp.lit(x) }

factory method bool(b:Boolean) -> Exp {

method x -> Boolean { b }

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factory method iff(c:Exp, t:Exp, f:Exp) -> Exp {

method eval -> Value {

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method pretty -> String {

 "if ({c.pretty}) then {t.pretty} else {f.pretty}"

 }

}

def e3plus4:Exp = sum(lit 3, lit 4)

def e2plus6:Exp = sum(lit 2, lit 6)

def ett:Exp = bool(**true**)

def ifExpr:Exp = iff(ett, e3plus4, e2plus6)

print "{ifExpr.pretty} = {ifExpr.eval}"

// prints: if (true) then 3 + 4 else 2 + 6 = 7

exp+pretty+bool.grace

```
...
3 + 4 = 7
if (true) then 3 + 4 else 2 + 6 = 7
$
```


Object Algebras

- Oliveira and Cook. “Extensibility for the masses”. ECOOP 2012
 - Avoids typing issues (beyond type parameters) and permits re-use of creation code.
 - Basic idea: abstract over creation by defining a method that builds the structure on demand
 - Argument to that method is the “Object Algebra” — a factory object


```
dialect "staticTypes"  
import "exp_base" as exp  
type Exp = exp.Exp  
  
// define the Object Algebra machinery  
type IntAlg<A> = {  
  lit(x:Number) -> A  
  sum(e1:A, e2:A) -> A  
}  
factory method intFactory -> IntAlg<Exp> {  
  method lit(x:Number) -> Exp { exp.lit(x) }  
  method sum(a:Exp, b:Exp) -> Exp { exp.sum(a, b) }  
}  
method mk3Plus4<A>(v:IntAlg<A>) -> A {  
  v.sum(v.lit(3), v.lit(4))  
}  
  
// compare the above with the normal expression:  
// def e3Plus4:Exp = sum(lit 3, lit 4)
```


// add pretty—printing to expressions "retroactively"

```
type Pretty = { pretty -> String }  
factory method prettyFactory -> IntAlg<Pretty> {  
  factory method lit(x:Number) {  
    method pretty -> String { x.asString }  
  }  
  factory method sum(a:Pretty, b:Pretty) {  
    method pretty -> String { "{a.pretty} + {b.pretty}" }  
  }  
}
```

objectAlgebra.grace (2)

// demonstration

```
def x = mk3Plus4(intFactory)  
// print "{x.pretty} = {x.eval}"  
// fails: no method 'pretty' in object x  
def s = mk3Plus4(prettyFactory)  
// print "{s.pretty} = {s.eval}"  
// fails: no method 'eval' in object s  
print "{s.pretty} = {x.eval}"  
// prints: 3 + 4 = 7
```


// add pretty—printing to expressions "retroactively"

```
type Pretty = { pretty -> String }  
factory method prettyFactory -> IntAlg<Pretty> {  
  factory method lit(x:Number) {  
    method pretty -> String { x.asString }  
  }  
  factory method sum(a:Pretty, b:Pretty) {  
    method pretty -> String { "{a.pretty} + {b.pretty}" }  
  }  
}
```

objectAlgebra.grace (2)

// demonstration

```
def x = mk3Plus4(intFactory)  
// print "{x.pretty} = {x.eval}"  
// fails: no method 'pretty' in object x  
def s = mk3Plus4(prettyFactory)  
// print "{s.pretty} = {s.eval}"  
// fails: no method 'eval' in object s  
print "{s.pretty} = {x.eval}"  
// prints: 3 + 4 = 7
```

```
...  
3 + 4 = 7  
$
```


Independent Extensibility

In real life, a much more common scenario than Fig. 1 followed by Fig. 2 followed by Fig. 3 would be like this. Some party *A* defines *exp_and_pretty*. Another party *B* independently defines *exp_and_bool*. A third party *C* finds those and wants to combine them to *exp_and_pretty_and_bool*. This should be possible so that *C* need only define *pretty* for *bool* (in addition to importing the two previous modules). Can Grace handle that?

- Adding *pretty* uses inheritance, while adding *bool* uses composition.
 - If both the original extensions used inheritance, we couldn't guarantee that we could combine them

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Yes!

- But solution is not fully general

- Adding *pretty* uses inheritance, while adding *bool* uses composition.
 - If both the original extensions used inheritance, we couldn't guarantee that we could combine them

Conclusions

- Wadler's version of the expression problem is unsolvable
- Wadler saw it as a challenge for type systems
- I see it as a challenge for even more fundamental features of a language:
 - global constants *vs* local namespaces
 - presence of built-in “non-objects”,
 - client object creation with method request or primitive