

# Computational Photography

---

**Prof. Feng Liu**

**Spring 2022**

<http://www.cs.pdx.edu/~fliu/courses/cs510/>

**04/21/2022**

# Last Time

---

- Re-lighting
  - HDR

# Today

---

- Panorama
  - Overview
  - Feature detection

# Panorama Building: History

---



*Along the River During Ching Ming Festival*  
by Z.D Zhang (1085-1145 )

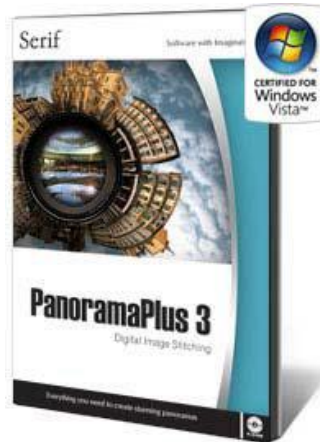
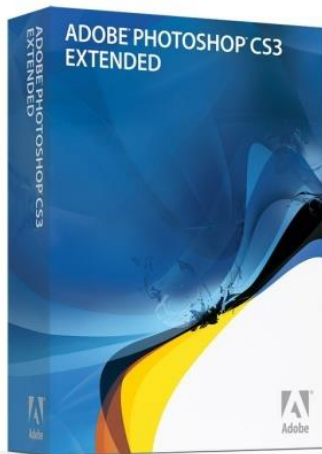


*San Francisco from Rincon Hill, 1851,*  
by Martin Behrmanx

# Panorama Building: A Concise History

---

- The state of the art and practice is good at assembling images into panoramas



- Mid 90s -Commercial Players (e.g. QuicktimeVR)
- Late 90s -Robust stitchers (in research)
- Early 00s -Consumer stitching common
- Mid 00s -Automation

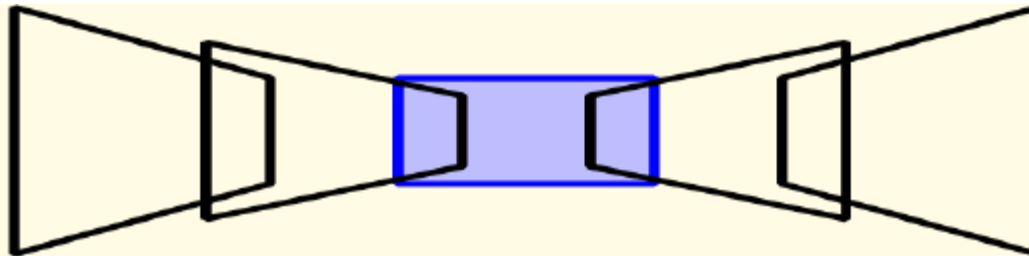
# Stitching Recipe

---

- Align pairs of images



- Align all to a common frame



- Adjust (Global) & Blend



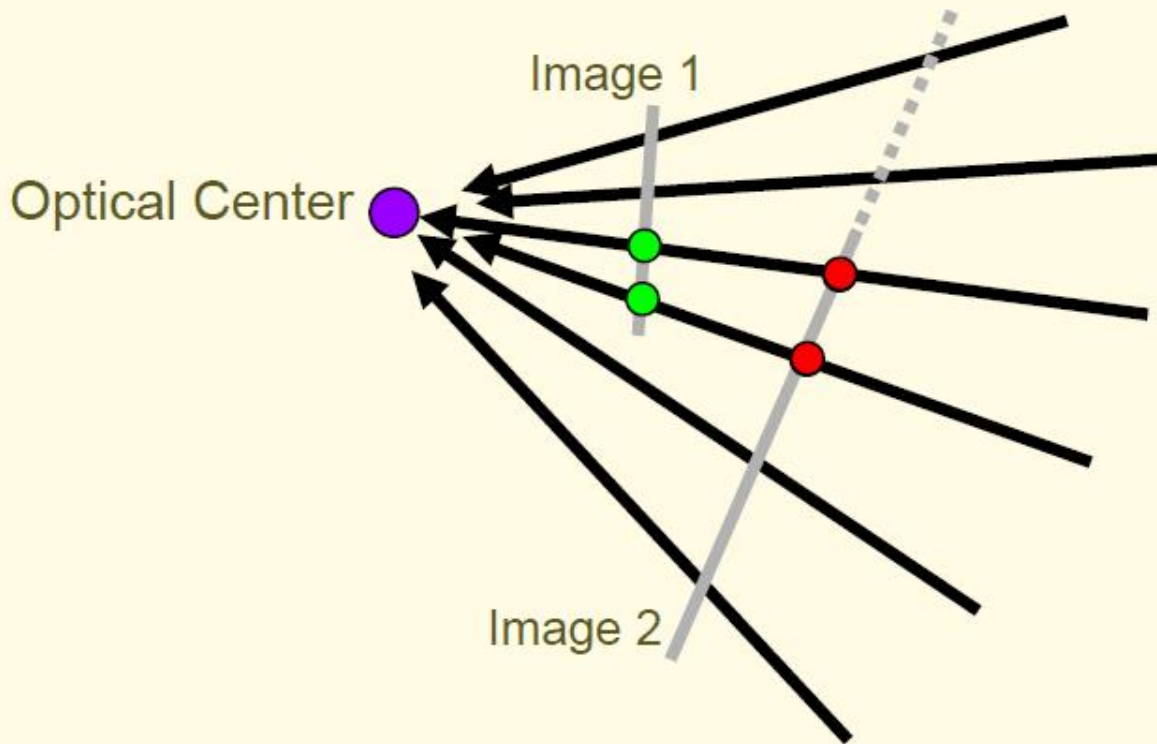
# Stitching Images Together

---



# When do two images “stitch”?

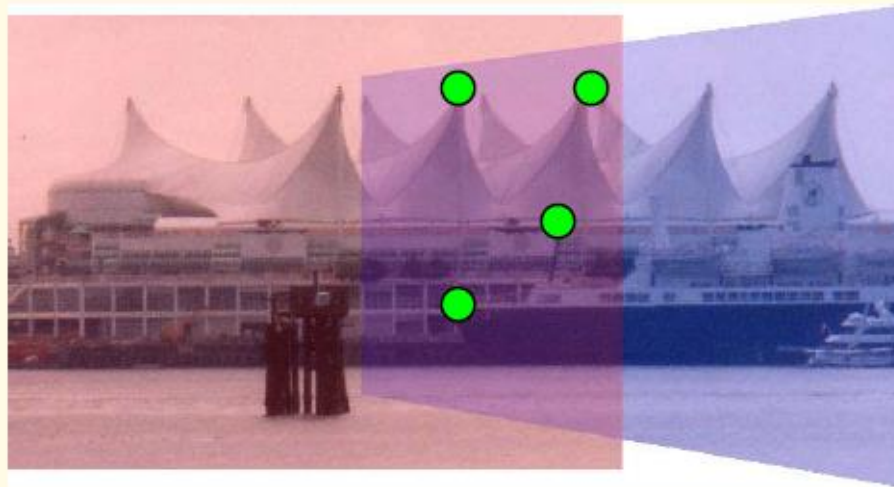
Images taken from the same viewpoint are related





# Images can be transformed to match

---

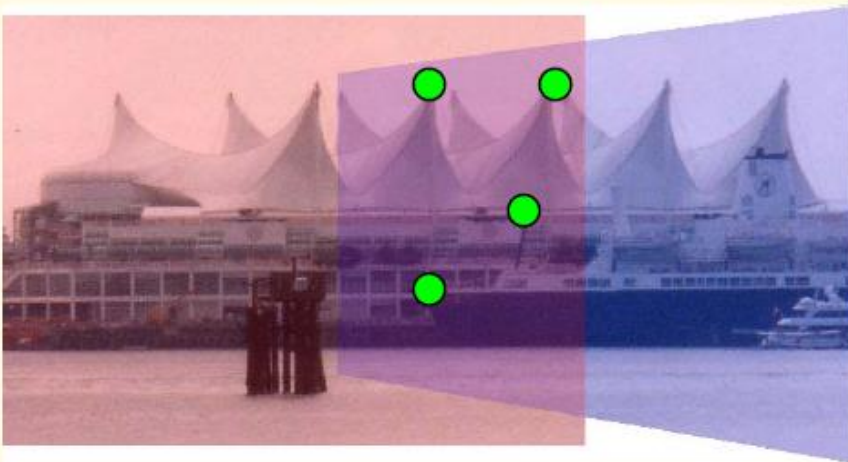


# Images are related by *Homographies*

- 8 parameter, 2D Image Transformation

$$x', y' = \frac{ax + by + c}{gx + hy + 1}, \frac{dx + ey + f}{gx + hy + 1}$$

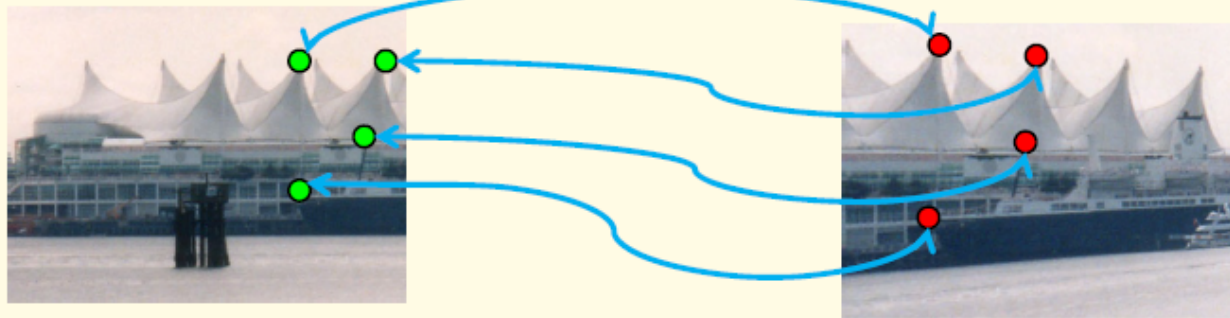
$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



# Compute Homographies

---

- Find Corresponding Features\*
- Compute Best-Fit Homography  
(using robust statistics e.g. RANSAC)

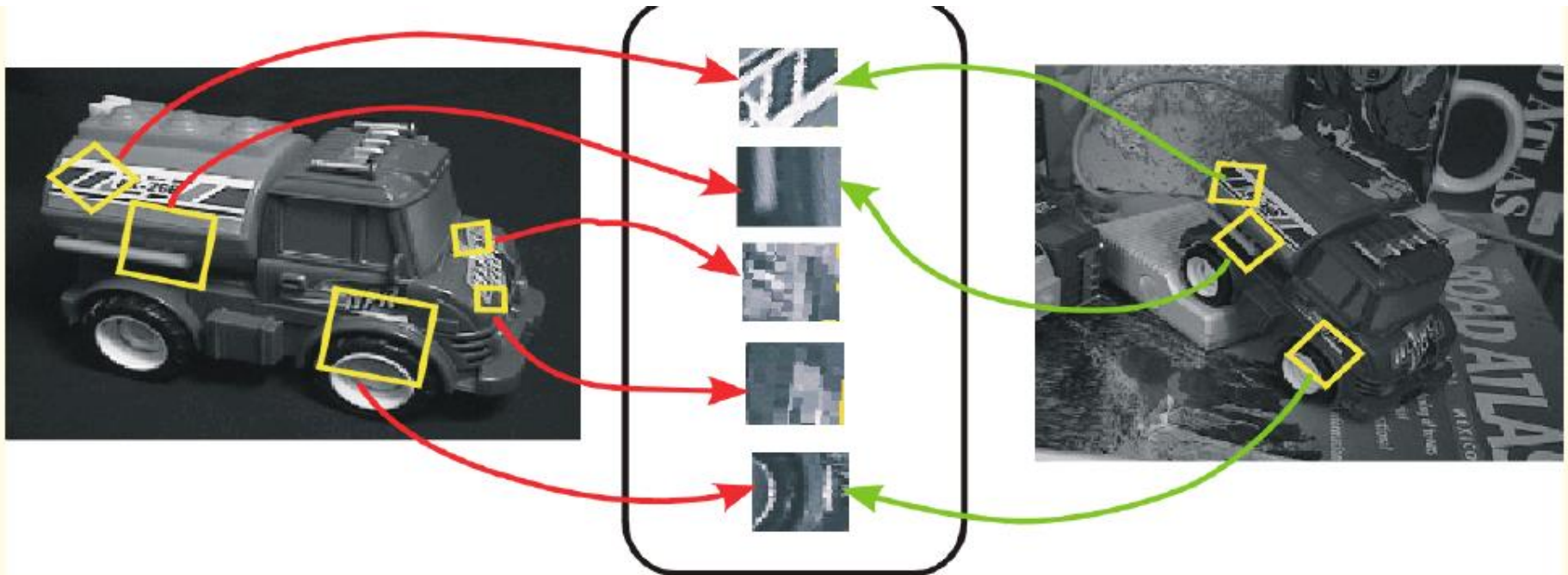


- Two images stitch if and only if the best fit homography is a good fit
- If the best fit homography is a bad fit, the resulting panorama will be bad.

# Automatic Feature Points Matching

---

- ❑ Match local neighborhoods around points
- ❑ Use descriptors to efficiently compare SIFT
  - [Lowe 04] most common choice



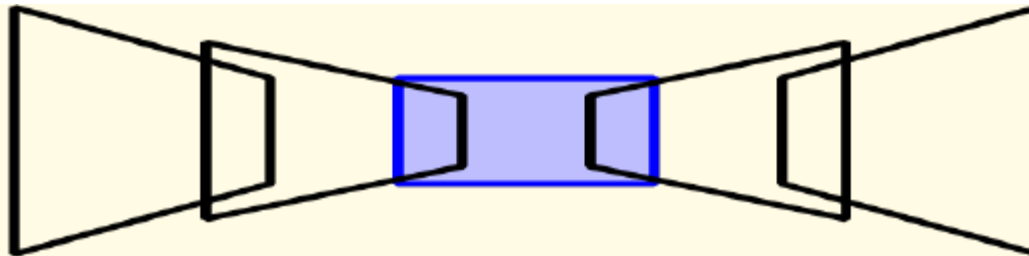
# Stitching Recipe

---

- Align pairs of images



- Align all to a common frame



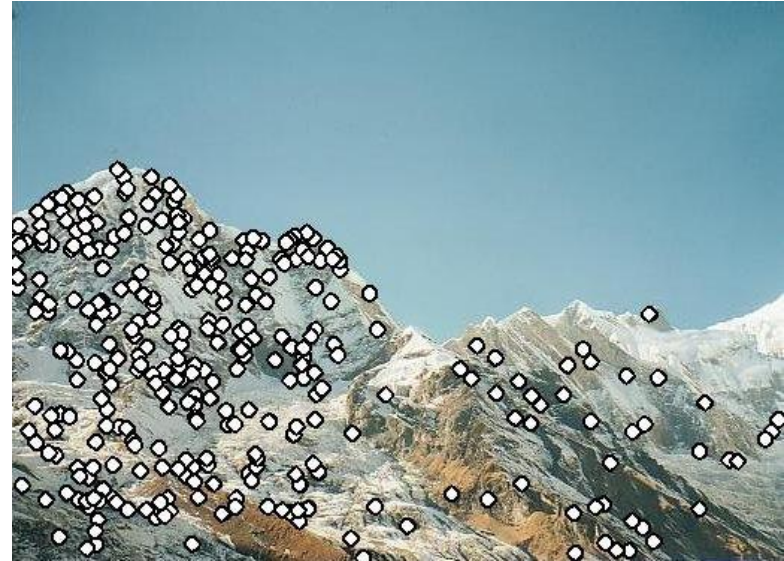
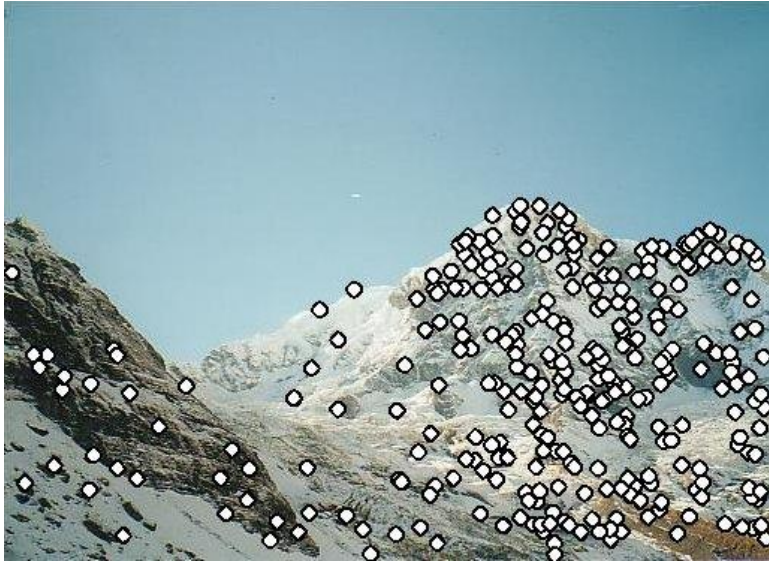
- Adjust (Global) & Blend



# Wide Baseline Matching

---

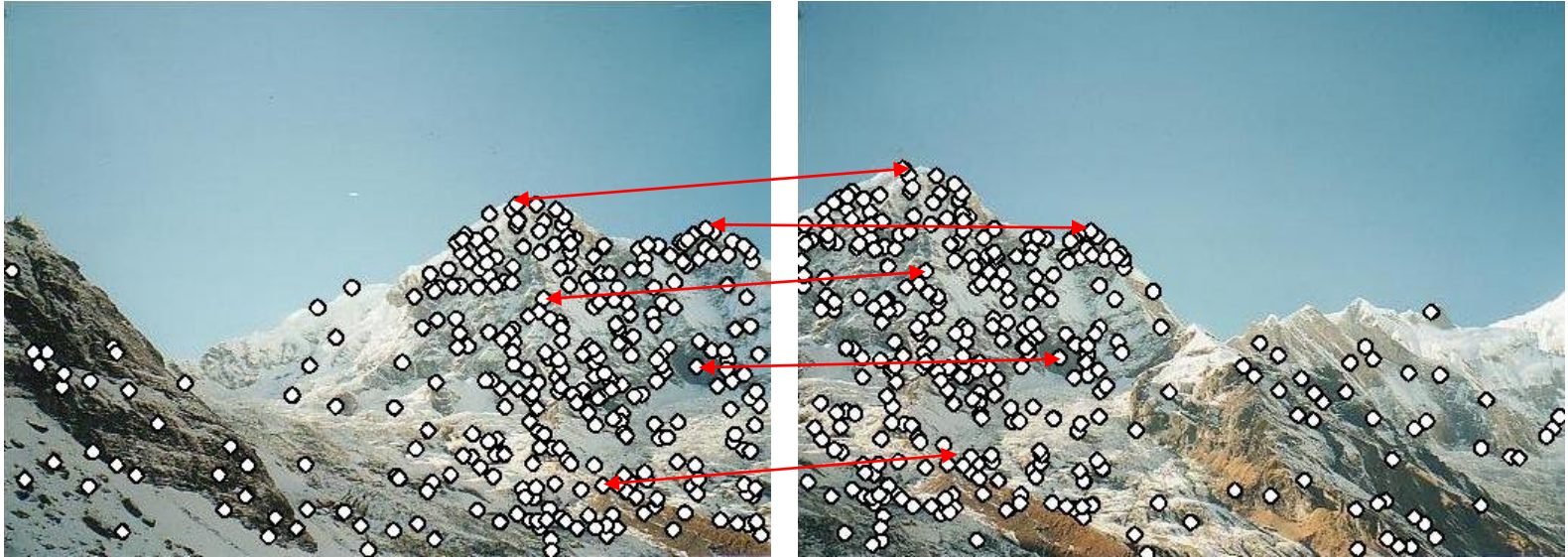
- Images taken by cameras that are far apart make the correspondence problem very difficult
- Feature-based approach: Detect and match feature points in pairs of images



# Matching with Features

---

- Detect feature points
- Find corresponding pairs



# Matching with Features

---

□ Problem 1:

- Detect the *same* point *independently* in both images



**no chance to match!**

**We need a repeatable detector**

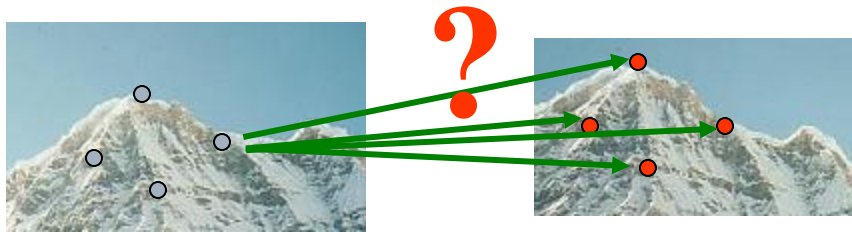


# Matching with Features

---

□ Problem 2:

- For each point correctly recognize the corresponding point



We need a reliable and distinctive **descriptor**

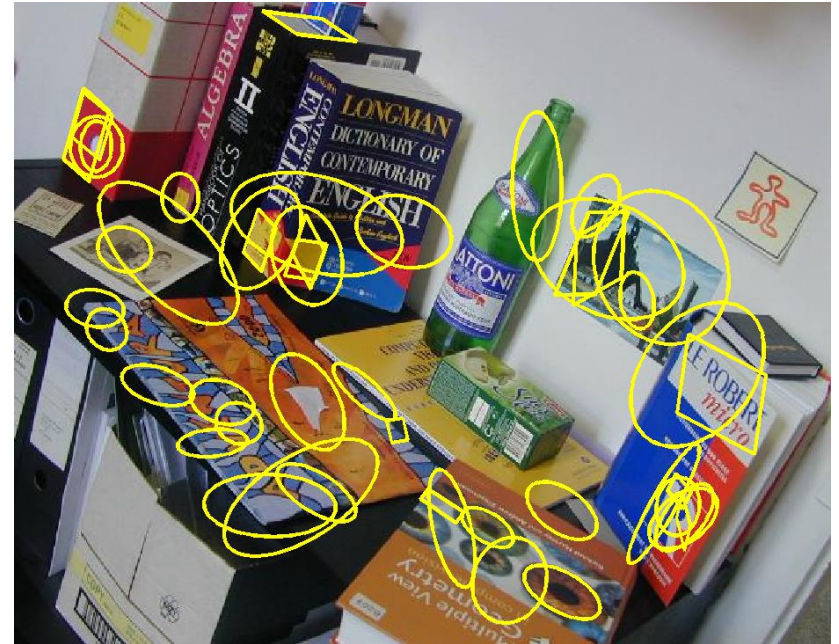
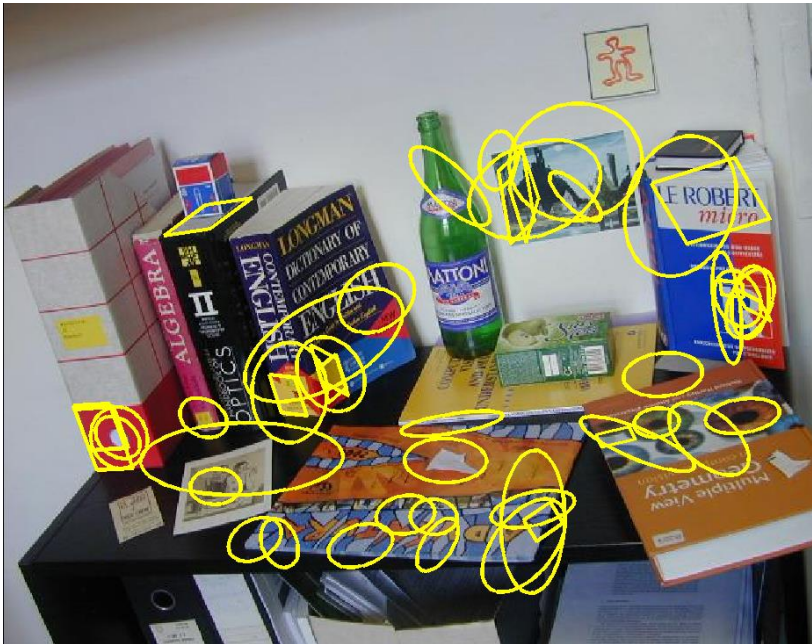
# Properties of an Ideal Feature

---

- ❑ **Local:** features are local, so robust to occlusion and clutter (no prior segmentation)
  - ❑ **Invariant** (or covariant) to many kinds of geometric and photometric transformations
  - ❑ **Robust:** noise, blur, discretization, compression, etc. do not have a big impact on the feature
  - ❑ **Distinctive:** individual features can be matched to a large database of objects
  - ❑ **Quantity:** many features can be generated for even small objects
  - ❑ **Accurate:** precise localization
  - ❑ **Efficient:** close to real-time performance
-

# Problem 1: Detecting Good Feature Points

---



[Image from T. Tuytelaars ECCV 2006 tutorial]

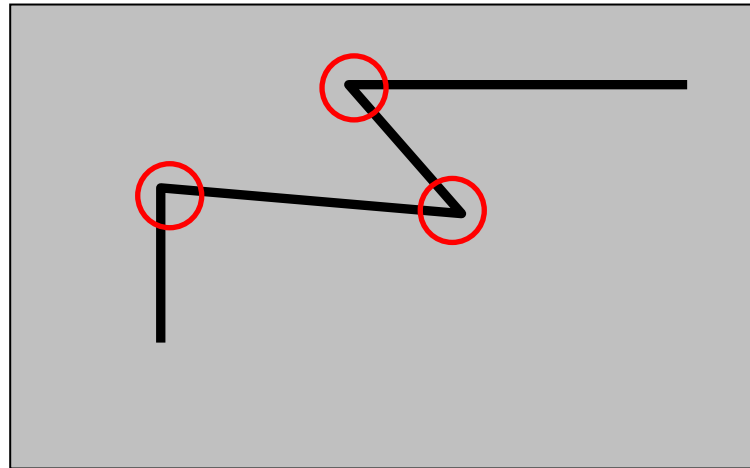
# Feature Detectors

---

- Hessian
- Harris
- Lowe: SIFT (DoG)
- Mikolajczyk & Schmid:  
Hessian/Harris-Laplacian/Affine
- Tuytelaars & Van Gool: EBR and IBR
- Matas: MSER
- Kadir & Brady: Salient Regions
- Others

# Harris Corner Point Detector

---



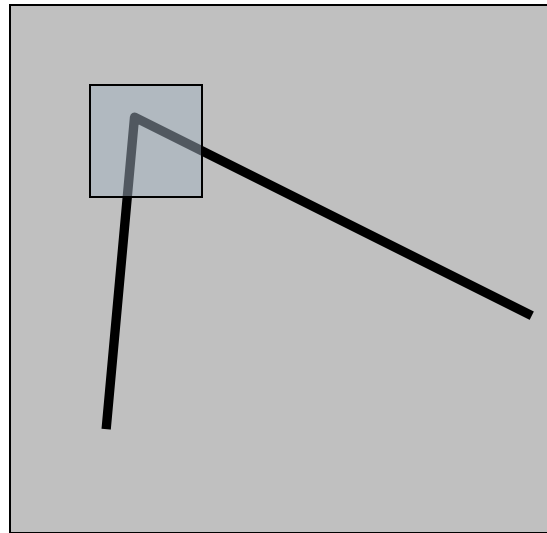
C. Harris, M. Stephens, "A Combined Corner and Edge Detector," 1988

---

# Harris Detector: Basic Idea

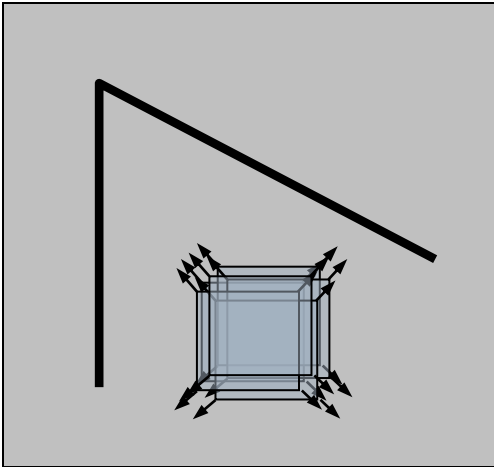
---

- We should recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in response

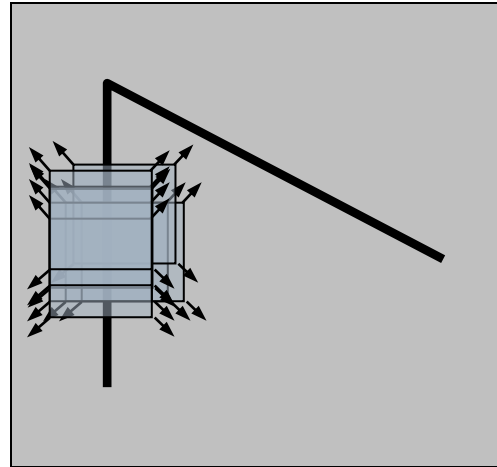


# Harris Detector: Basic Idea

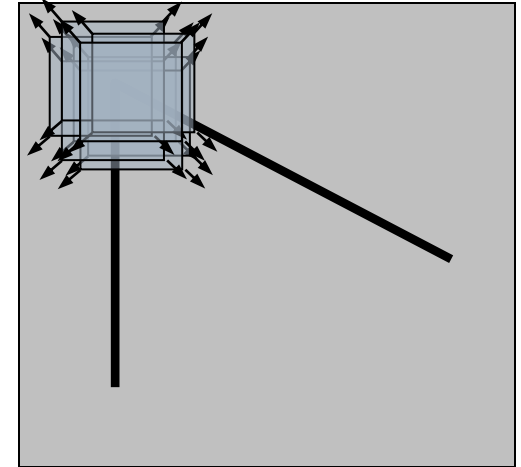
---



“flat” region:  
no change in  
all directions



“edge”:  
no change along  
the edge direction



“corner”:  
significant change  
in *all* directions

# Harris Detector: Derivation

Change of intensity for a (small) shift by  $[u, v]$  in image  $I$ :

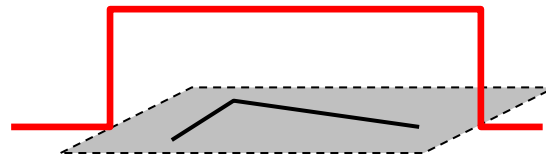
$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Weighting  
function

Shifted  
intensity

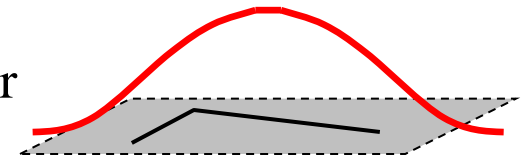
Intensity

Weighting function  $w(x, y) =$



1 in window, 0 outside

or



Gaussian



# Calculus: Taylor Series Expansion

---

A real function  $f(x+u)$  can be approximated as the 2<sup>nd</sup> order of its Taylor series expansion at a point  $x$ .

$$f(x + u) = f(x) + uf'(x) + O(u^2)$$

# Derivatives

---

For 1D function  $f(x)$ , the derivative is:

$$\frac{\partial f(x)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon) - f(x)}{\varepsilon}$$

For 2D function  $f(x,y)$ , the partial derivative is:

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

# Derivatives

---

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x+1, y) - f(x, y)}{1}$$

To implement above as convolution, what would be the associated filter?

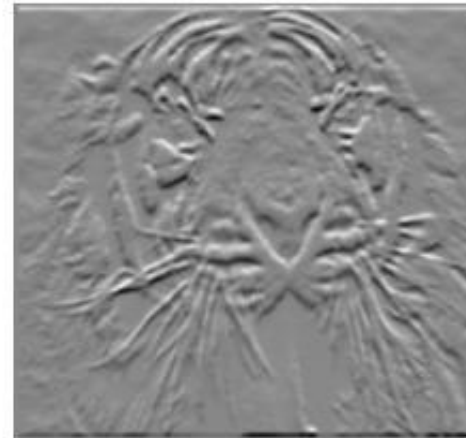
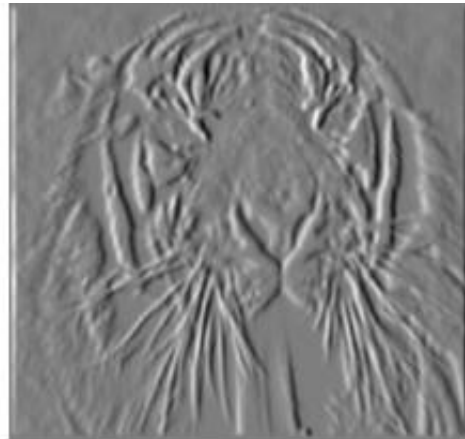
# Partial derivatives of an image

---



$$\frac{\partial f(x, y)}{\partial x}$$

|    |   |
|----|---|
| -1 | 1 |
|----|---|



$$\frac{\partial f(x, y)}{\partial y}$$

|    |    |    |
|----|----|----|
| -1 | or | 1  |
| 1  |    | -1 |

Which shows changes with respect to x?

# Finite difference filters

---

Other approximations of derivative filters exist:

**Prewitt:**  $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

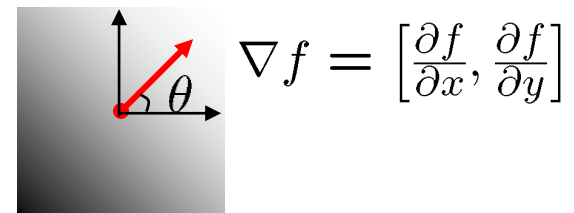
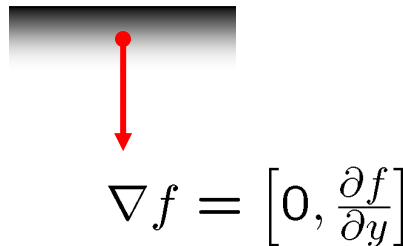
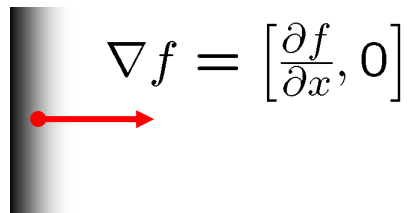
**Sobel:**  $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

**Roberts:**  $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

# Image gradient

---

The gradient of an image:  $\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$



The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

# Harris Detector

---

Apply 2<sup>nd</sup> order Taylor series expansion:

$$\begin{aligned} E(u, v) &= \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2 \\ &= \sum_{x, y} w(x, y) [I_x u + I_y v + O(u^2, v^2)]^2 \end{aligned}$$

$$E(u, v) = Au^2 + 2Cuv + Bv^2$$

$$A = \sum_{x, y} w(x, y) I_x^2(x, y)$$

$$B = \sum_{x, y} w(x, y) I_y^2(x, y)$$

$$C = \sum_{x, y} w(x, y) I_x(x, y) I_y(x, y)$$

$$E(u, v) = \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & C \\ C & B \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$I_x = \partial I(x, y) / \partial x$$

$$I_y = \partial I(x, y) / \partial y$$

# Harris Corner Detector

---

Expanding  $E(u,v)$  in a 2<sup>nd</sup> order Taylor series, we have, for small shifts,  $[u, v]$ , a *bilinear* approximation:

$$E(u, v) \cong [u, v] \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $\mathbf{M}$  is a  $2 \times 2$  matrix computed from image derivatives:

$$\mathbf{M} = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad \begin{aligned} I_x &= \partial I(x, y) / \partial x \\ I_y &= \partial I(x, y) / \partial y \end{aligned}$$

Note: Sum computed over small neighborhood around given pixel

---



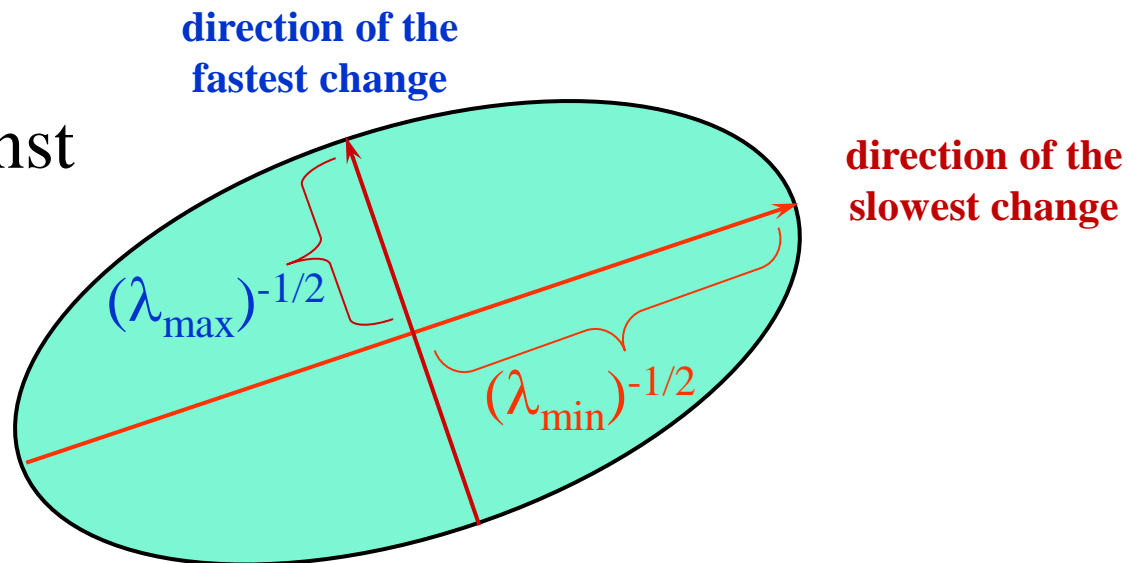
# Harris Corner Detector

---

Intensity change in shifting window: eigenvalue analysis

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } \mathbf{M}$$

Ellipse  $E(u, v) = \text{const}$

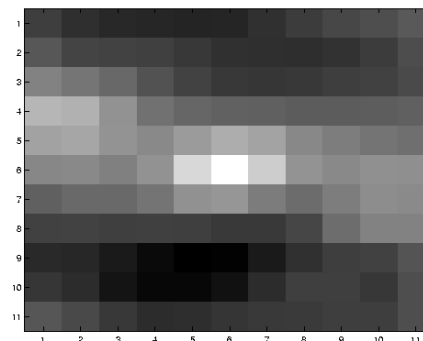


# Selecting Good Features

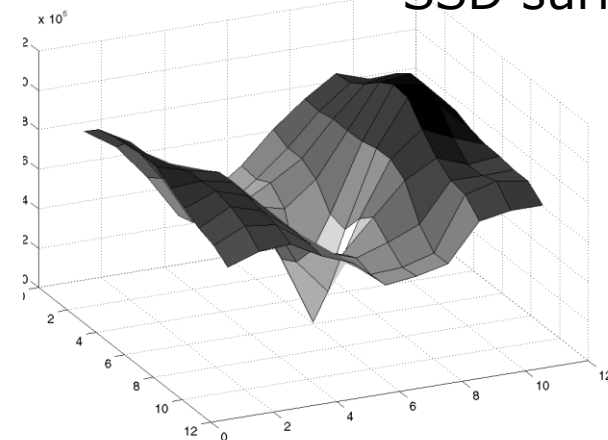
---



Image patch



SSD surface

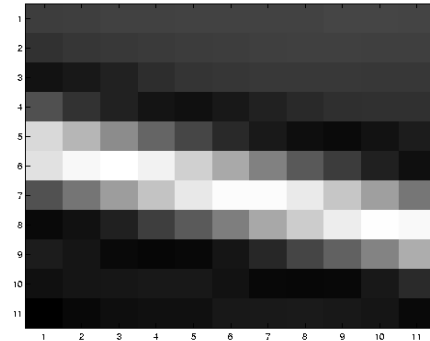


$\lambda_1$  and  $\lambda_2$  both large

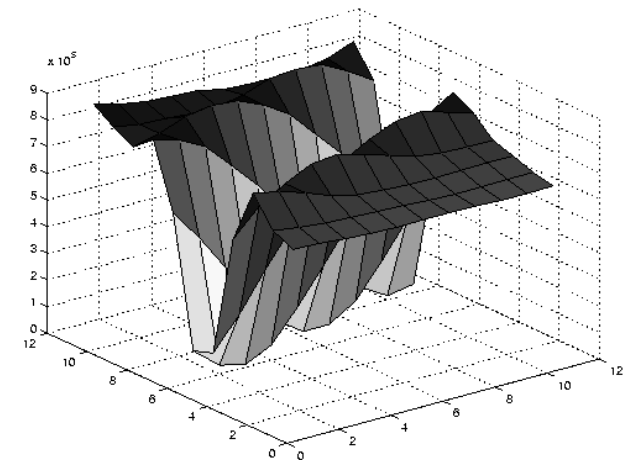
---

# Selecting Good Features

---



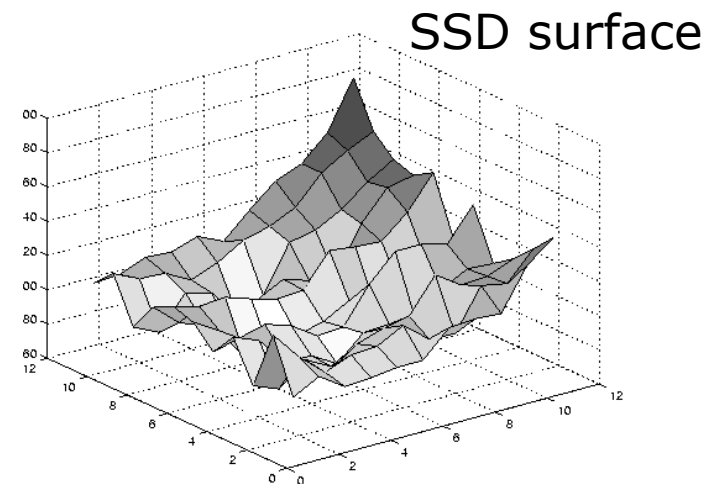
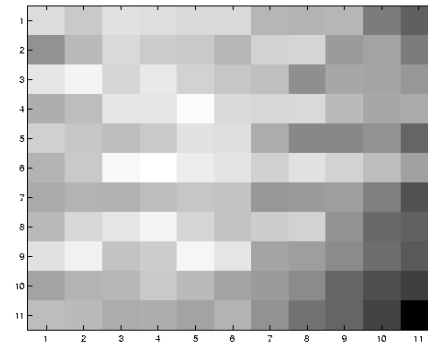
SSD surface



large  $\lambda_1$ , small  $\lambda_2$

# Selecting Good Features

---

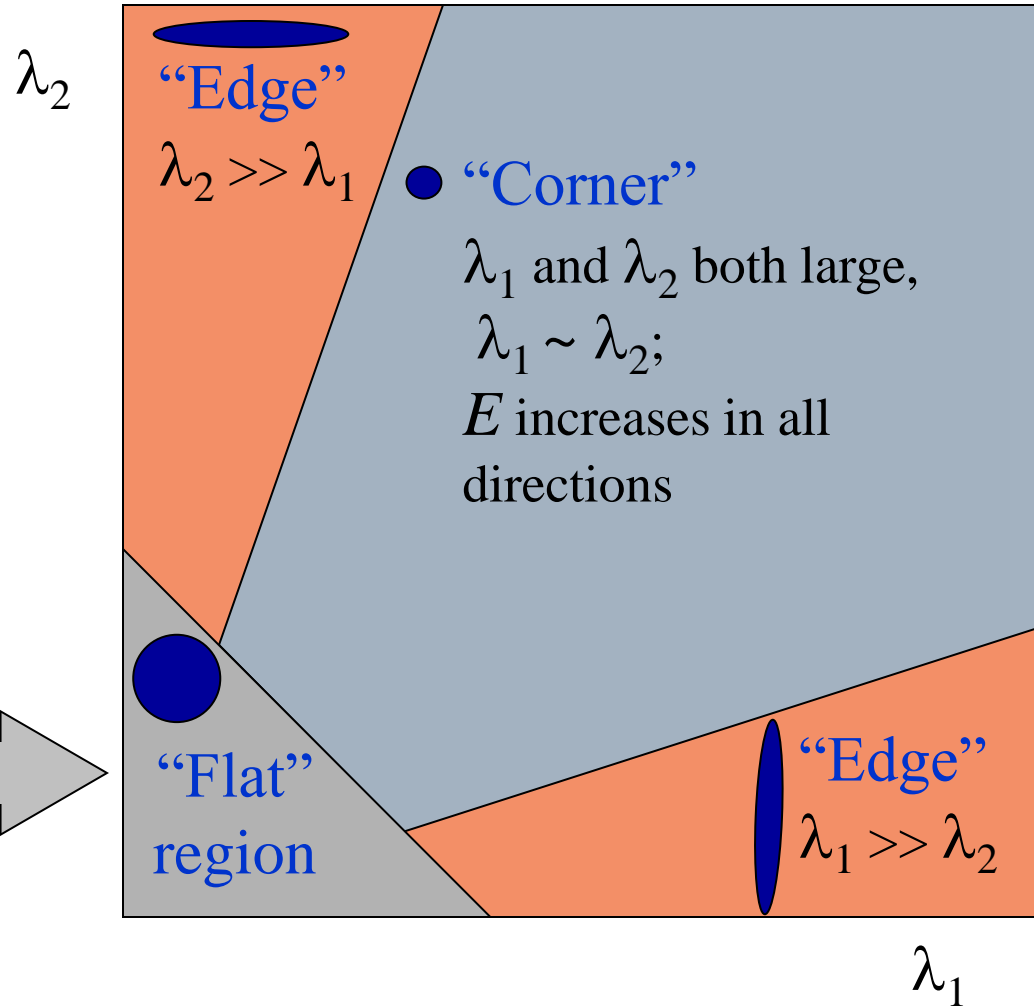


small  $\lambda_1$ , small  $\lambda_2$

# Harris Corner Detector

Classification of image points using eigenvalues of  $\mathbf{M}$ :

$\lambda_1$  and  $\lambda_2$  are small;  
 $E$  is almost constant  
in all directions



# Harris Corner Detector

---

Measure of corner response:

$$R = \det M - k (\text{trace } M)^2$$

$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

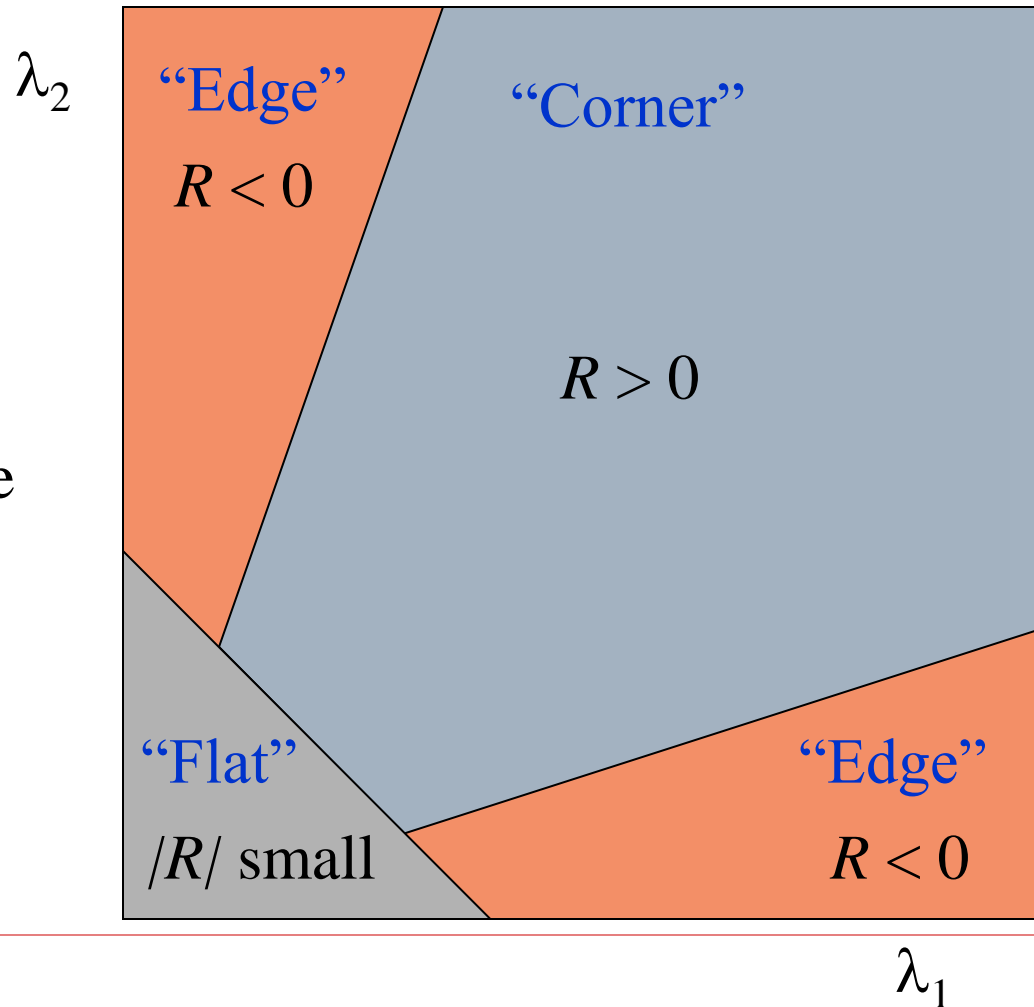
$k$  is an empirically-determined constant; e.g.,  $k = 0.05$

---

# Harris Corner Detector

---

- $R$  depends only on eigenvalues of  $\mathbf{M}$
- $R$  is large for a **corner**
- $R$  is negative with large magnitude for an **edge**
- $|R|$  is small for a **flat** region



# Harris Corner Detector: Algorithm

---

□ Algorithm:

1. Find points with large corner response function  $R$   
(i.e.,  $R > \text{threshold}$ )
2. Take the points of local maxima of  $R$  (for localization) by non-maximum suppression



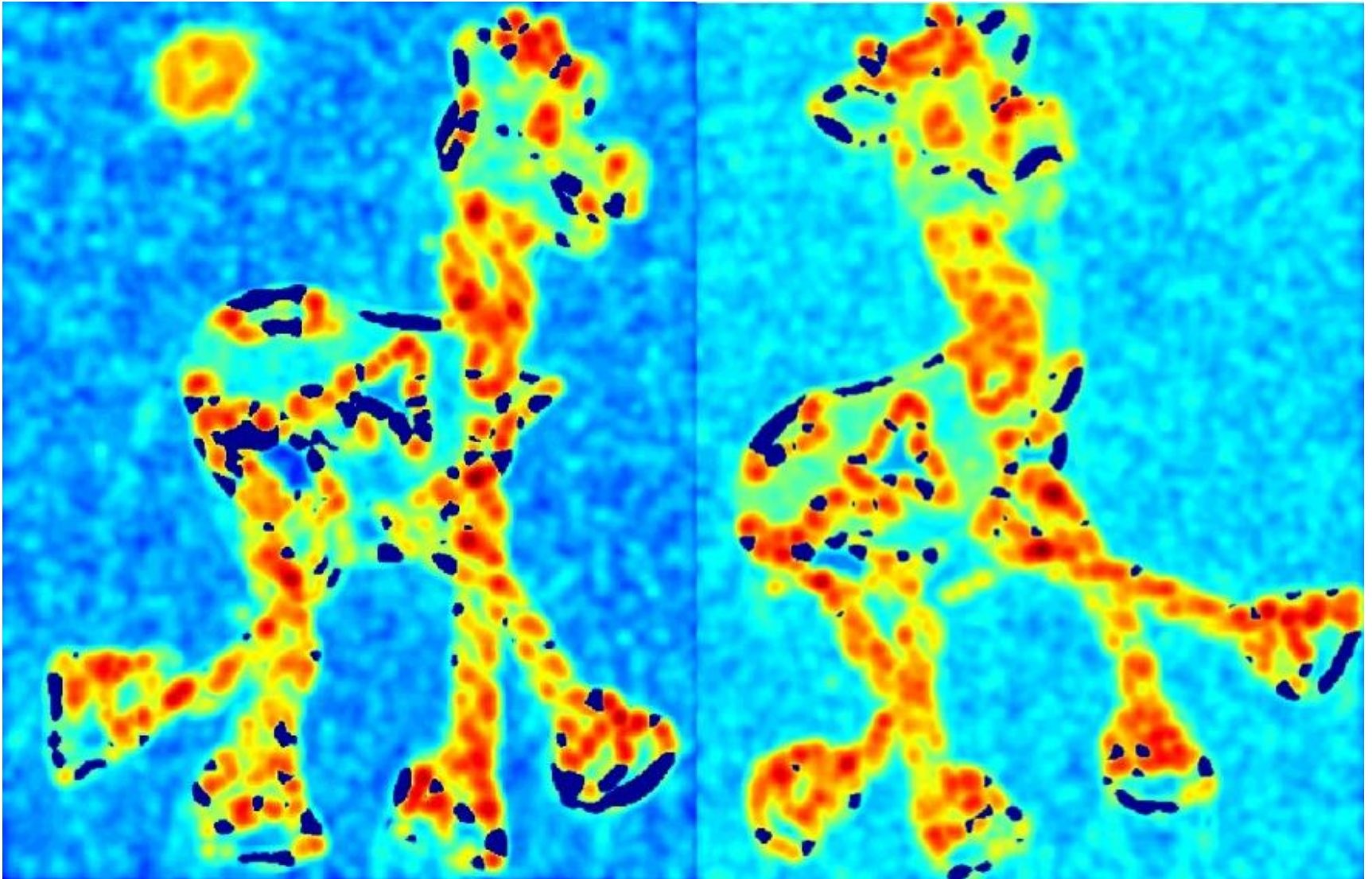
# Harris Detector: Example

---



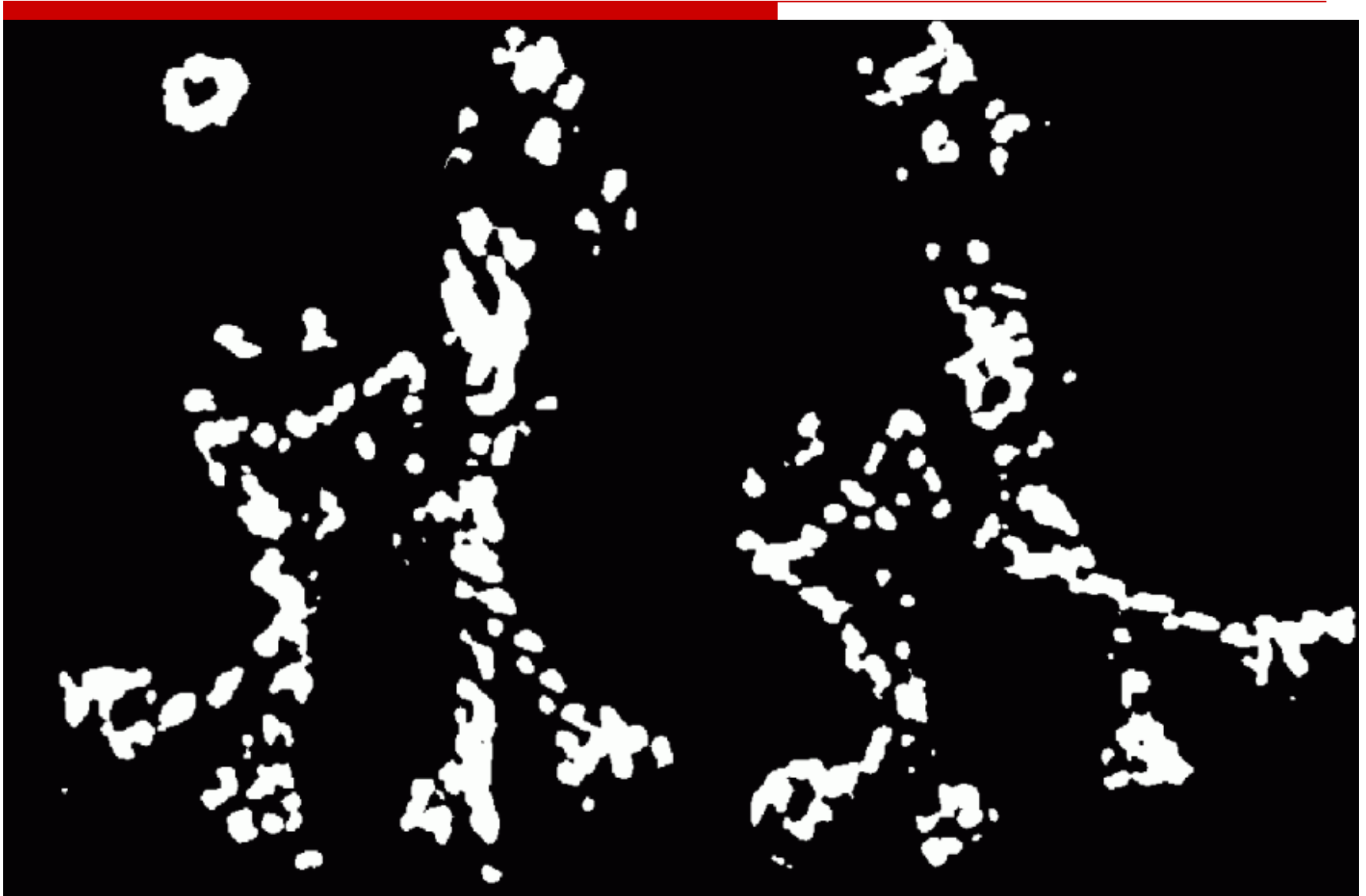
Credit: C. Dyer

# Harris Detector: Example



Credit: C. Dyer Compute corner response  $R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$

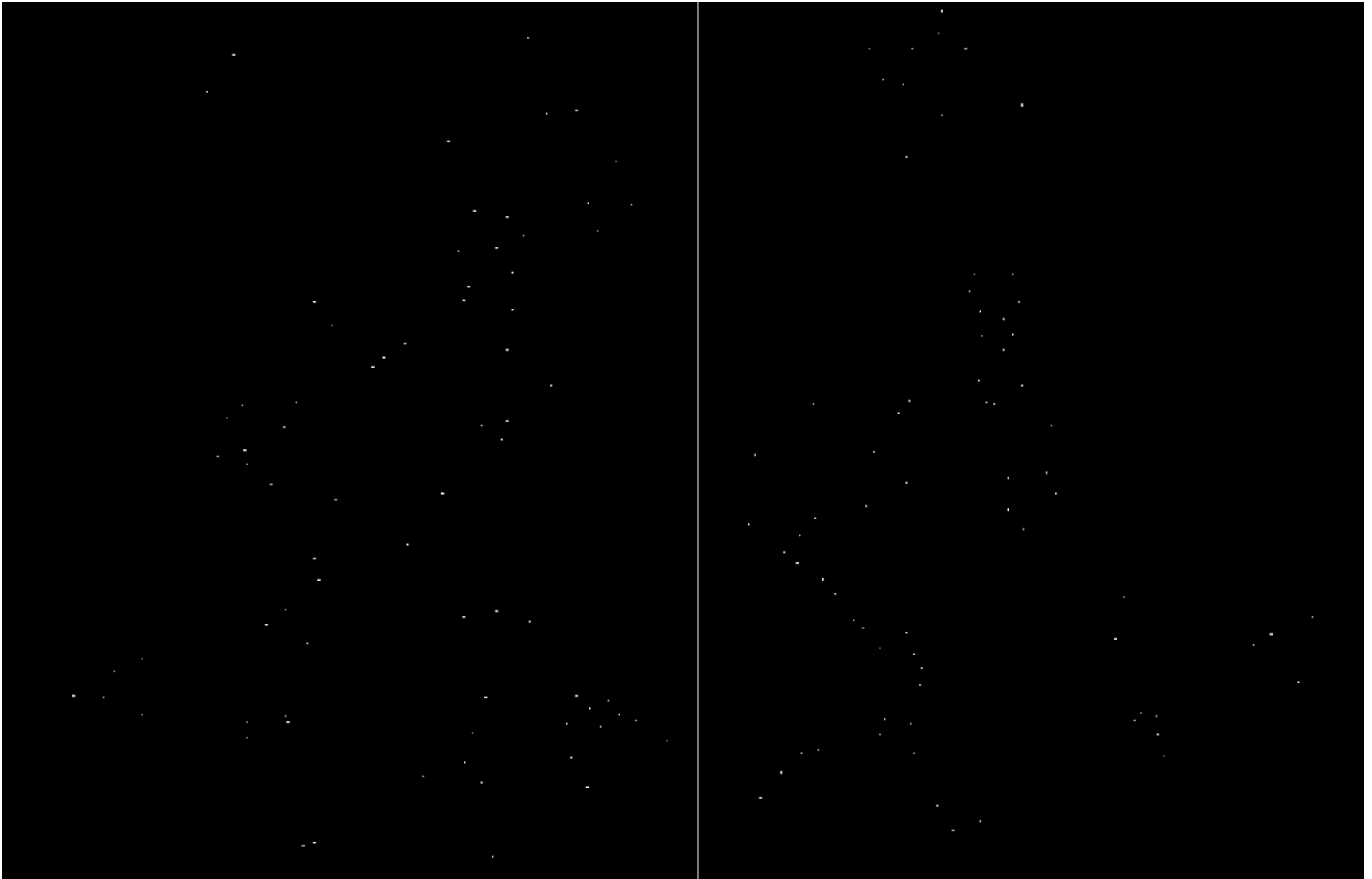
# Harris Detector: Example



Credit: C. Dyer Find points with large corner response:  $R > \text{threshold}$

# Harris Detector: Example

---



Credit: C. Dyer Take only the points of local maxima of  $R$

# Harris Detector: Example

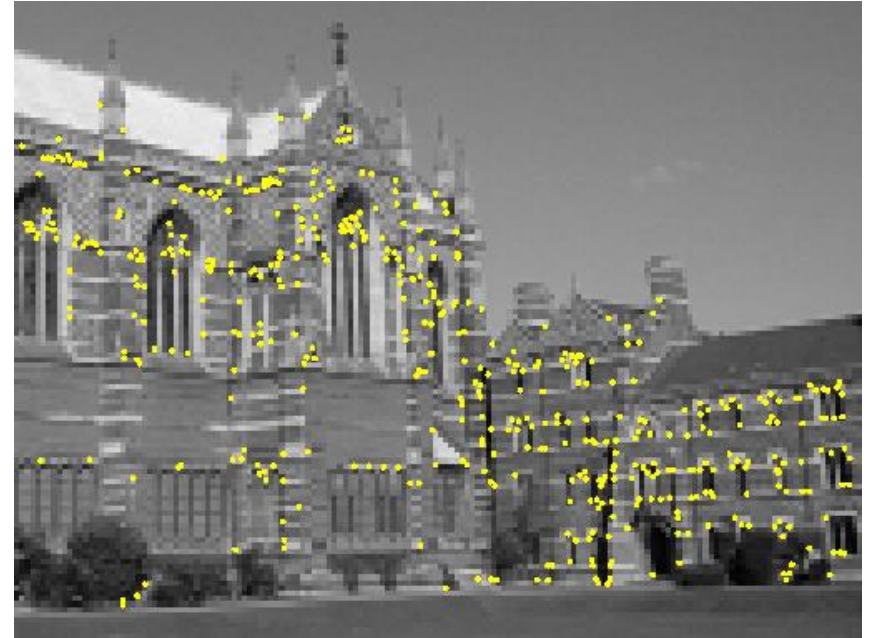
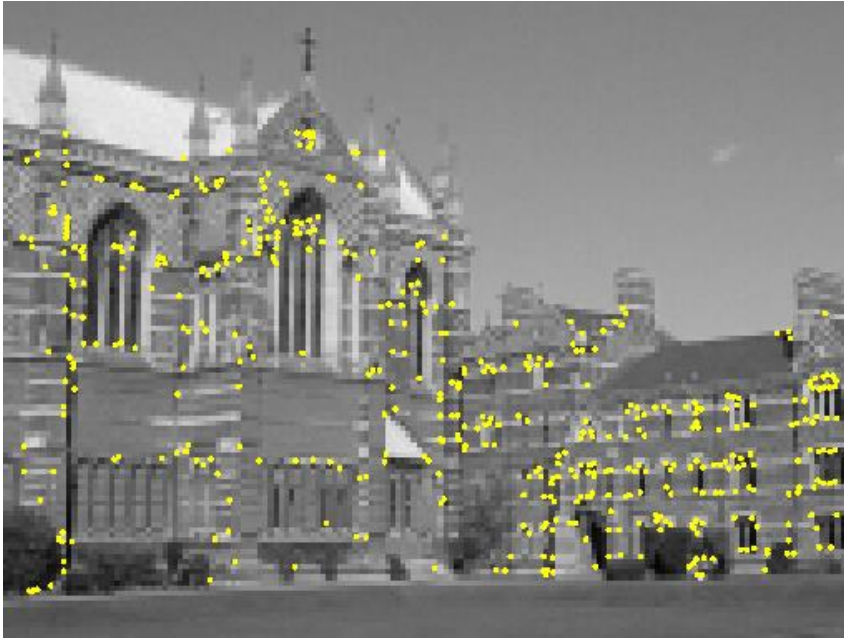
---



Credit: C. Dyer

# Harris Detector: Example

---

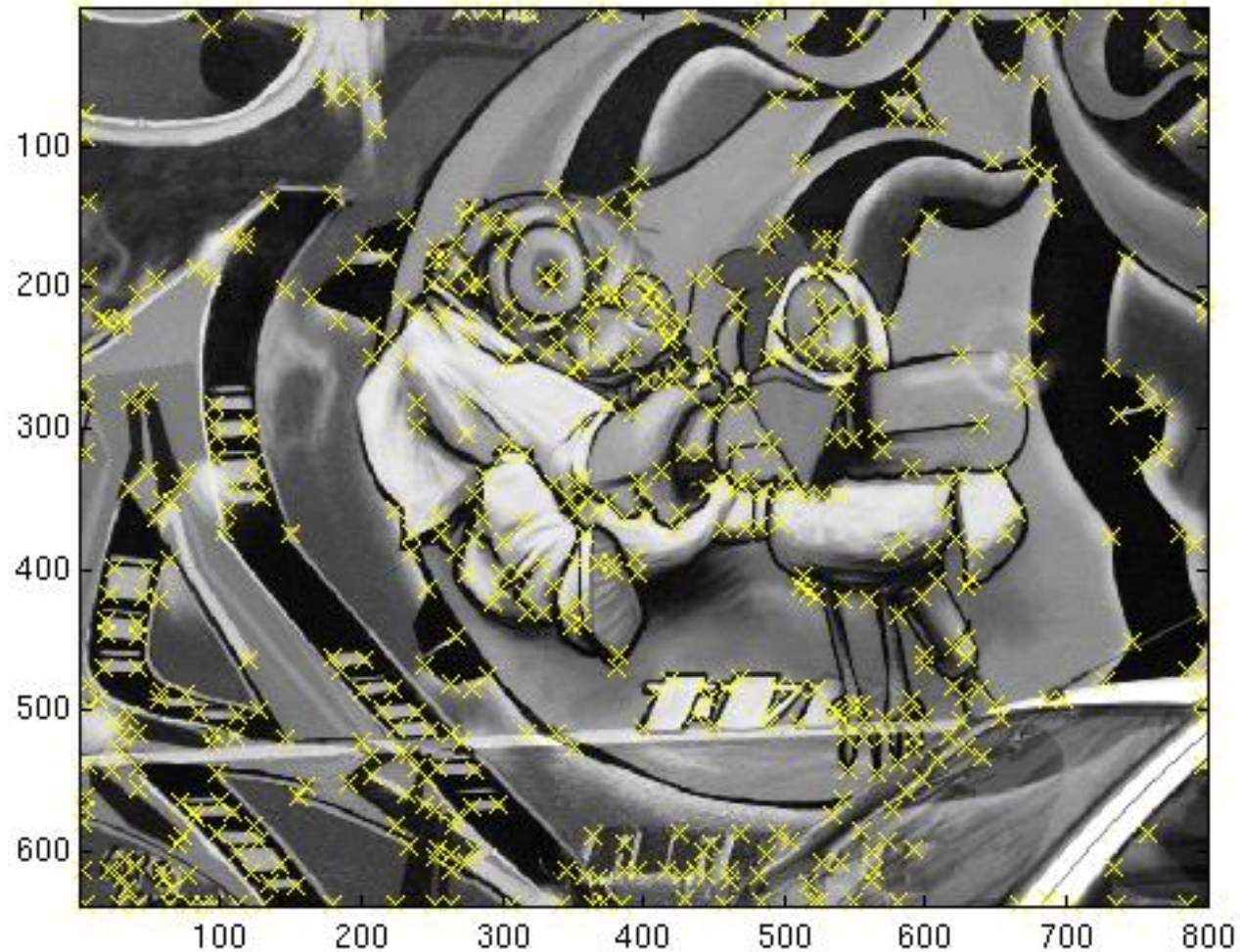


Interest points extracted with Harris ( $\sim 500$  points)

---

# Harris Detector: Example

---



# Harris Detector: Summary

---

- Average intensity change in direction  $[u, v]$  can be expressed in bilinear form:

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

- Describe a point in terms of eigenvalues of  $M$ :  
*measure of corner response:*

$$R = \lambda_1 \lambda_2 - k (\lambda_1 + \lambda_2)^2$$

- A good (corner) point should have a *large intensity change in all directions*, i.e.,  $R$  should be a large positive value



# Student paper presentation

---

## Color harmonization

D. Cohen-Or, O. Sorkine, R. Gal, T. Leyv, and, Y. Xu  
ACM SIGGRAPH 2006

**Presenter:** Gerendasy, Daniel R

# Student paper presentation

---

## Colorization Using Optimization

A. Levin, D. Lischinski, and Y. Weiss  
ACM SIGGRAPH 2004

**Presenter:** Nguyen, Henry

# Next Time

---

- Panorama

- Feature and matching

- Student paper presentations

- 04/26: Li, Chi G

- Color Conceptualization

X. Hou and L. Zhang, ACM Multimedia 2007

- 04/26: Liu, Jiawei

- Photographic tone reproduction for digital images

E. Reinhard, M. Stark, P. Shirley, and J. Ferwerda,  
SIGGRAPH 2012