CHAPTER 0: PREREQUISITE KNOWLEDGE

SKIM THIS CHAPTER.
REVIEW AS NECESSARY.

SETS

$\{a, b, c\}$

$\mathbb{N}$ = Natural Numbers = $\{1, 2, 3, \cdots\}$

$\mathbb{Z}$ = Integers = $\{\ldots, -2, -1, 0, +1, +2, \cdots\}$

$\emptyset = \{\}$ empty set

$\cup$ Union

$\cap$ Intersection

$\overline{S}$ Complement

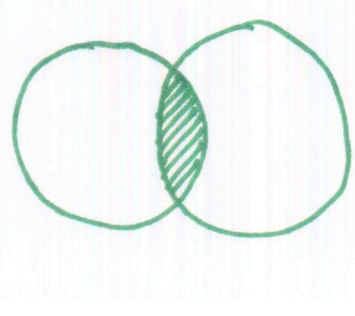
$S \times T$ Cross Product

$$\mathbb{N} \times \mathbb{N} = \{(i,j) \mid i, j \geq 1\}$$

$(4, 6)$ Tuples, Sequences

$P(S)$ Powerset

VENN DIAGRAMS



FUNCTIONS $f(a)$

f: Domain $\rightarrow$ Range

Unary, Binary functions

Arity, k-ary functions

Infix, Prefix

PROPERTY (or "PREDICATE")

P: Domain $\rightarrow$ {TRUE, FALSE}

RELATION $R: A \times A \times \ldots \times A \rightarrow$ {TRUE, FALSE}

REFLEXIVE: $\forall x.\ x R x$

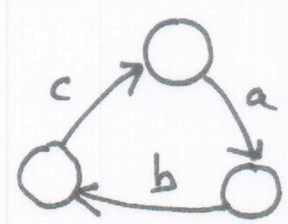
SYMMETRIC: $\forall x,y.\ xRy \Rightarrow yRx$

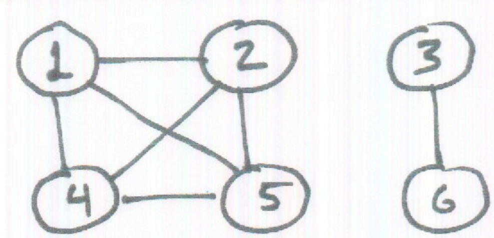
TRANSITIVE: $\forall x,y,z.\ xRy$ and yRz implies xRz

GRAPHS

EDGES AND VERTICES (or "Nodes")

$G=(V,E)$





DIRECTED / UNDIRECTED EDGES

LABELED / UNLABELED NODES

SUBGRAPHS, CONNECTED COMPONENTS

PATHS, CYCLES

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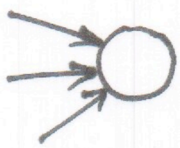
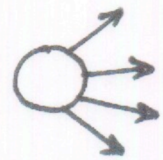
"IN-DEGREE"



"OUT-DEGREE"

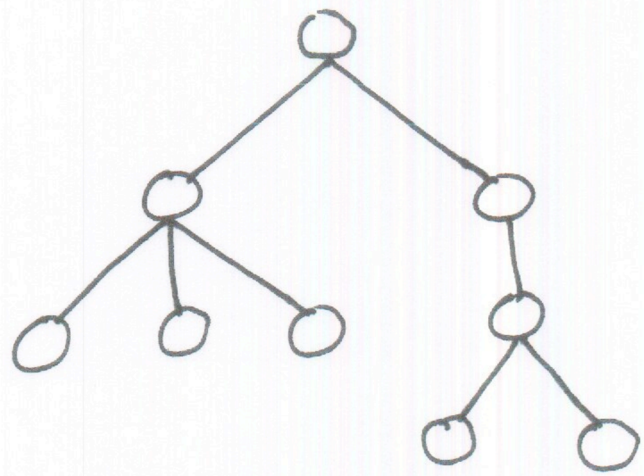


BINARY RELATION ≡ DIRECTED GRAPH

R(a, b) = TRUE

aRb

TREES

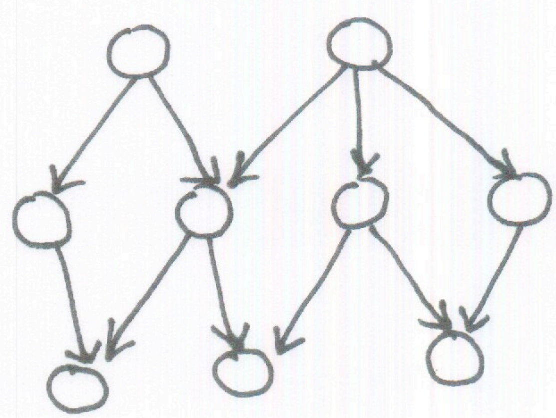


GRAPH WITH DIRECTED EDGES

NO CYCLES

ROOT NODE

DAG: Shared Parents Allowed



STRINGS

Σ = Alphabet = Set of symbols.

$$\Sigma = \{a, b, c, d\}$$

(Always a <u>finite</u> set!)

STRING = A <u>finite</u> sequence of symbols.

baccada

ϵ = Empty string = Epsilon (also ε)

Length of a string

$$w = baccada$$

$$|w| = |baccada| = 7$$

$$|\epsilon| = 0$$

xy = Concatenation

$x = bac$
$y = cada$
$xy = baccada$

x^R = Reverse of x

$x^R = cab$

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LANGUAGE

"A language is a set of strings."

$L_1 = \{ab, bc, ac, dd\}$

$L_2 = \{\epsilon, ab, abab, ababab, \ldots\}$

The empty string:

ϵ

The empty language:

$\{\} = \emptyset$

The language containing only ϵ:

$L_3 = \{\epsilon\}$

How to describe languages?

Enumeration $\{b, ac, da\}$

Regular Expressions $c(ab)^*(d|c)$

Context-Free Grammars

$S \to dTd$

$T \to TT \mid aTb \mid \epsilon$

Set Notation

$\{x \mid x \in \ldots\}$

Boolean Logic

And	$\wedge$	Conjunction
Or	$\vee$	Disjunction
Not	$\neg$	Negation (Also: $\overline{P}$)
	$\oplus$	Exclusive-Or
	$\leftrightarrow$	Equality (Also: $\Leftrightarrow$, $=$)
	$\rightarrow$	Implies, Implication (Also: $\Rightarrow$)

Distribution Laws:

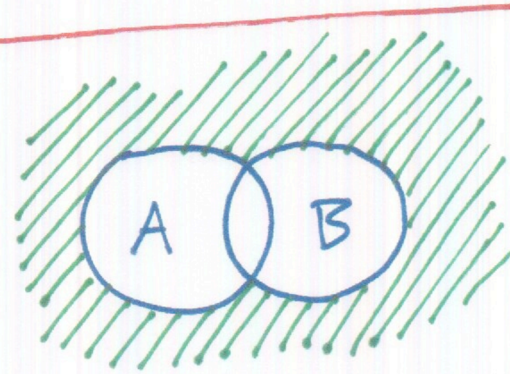
$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

DeMorgan's Laws

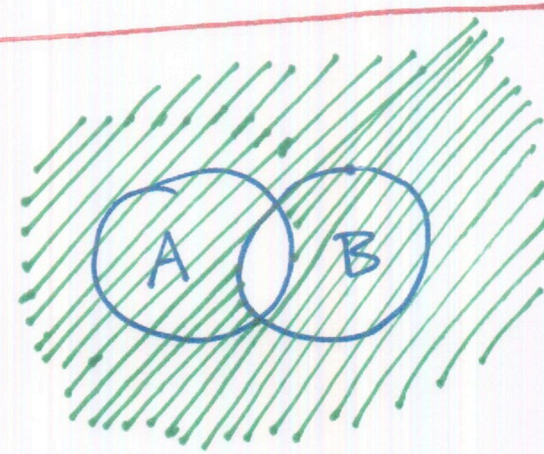
$$\neg(A \vee B) = (\neg A) \wedge (\neg B)$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$



$$\neg(A \wedge B) = (\neg A) \vee (\neg B)$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$



FIRST-ORDER LOGIC

FROM

$$\forall x. (\text{IsMan}(x) \Rightarrow \text{IsMortal}(x))$$

$$\text{IsMan}(\text{Socrates})$$

CONCLUDE

$$\text{IsMortal}(\text{Socrates})$$

Universe of Objects
Relations between objects
Statements are TRUE or FALSE
WFF: Well-formed formulas

FOR ALL $\forall$

$$\forall x. (\dots)$$

THERE EXISTS $\exists$

$$\exists x. (\dots)$$

Logical Operators

$$\forall x. (P(x) \rightarrow (\exists y. (R(x) \wedge P(y))))$$

Predicates

$$\exists y. \exists x. (P(f(x)) \vee P(g(f(y))))$$

functions

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DEFINITIONS

THEOREMS

Known to be true.

PROOFS

Mathematical arguments

Sometimes brief, sometimes elaborate

LEMMAS

Proved in isolation.

Part of a larger proof.

COROLLARY

A true statement (theorem).

Derived easily from the main theorem.

CONJECTURE

Unproven; Possibly true.

If-and-only-if

P iff Q (Also: $P \Leftrightarrow Q$)

Must Prove forward direction.

$P \Rightarrow Q$ "P only if Q"

Must Prove reverse direction.

$Q \Rightarrow P$ "P if Q"

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PROOF BY CONSTRUCTION

THEOREM: "x exists; There is an x."

PROOF: Show how to build an x.

PROOF BY CONTRADICTION

THEOREM: "P is true."

PROOF:

- Assume P is false.
- Do some logical reasoning.
- Conclude the truth of something known to be false (an "ABSURDITY").

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PROOF BY INDUCTION

THEOREM: "P is true for all... [integers ≥ 0]."

PROOF: BASIS CASE:

Show $P(0)$ is true.

INDUCTIVE STEP:

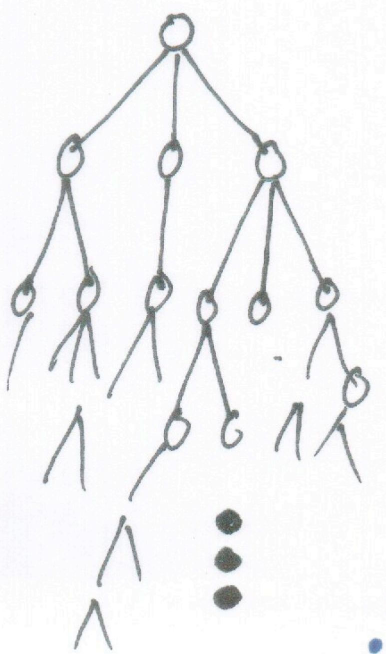
Assume $P(i)$ is true.

↑ The "inductive hypothesis"

Use logical reasoning to show $P(i+1)$ is true.

∴ Conclude P is true for all $i \geq 0$.

Structural Induction



BASIS CASE:

Show P is true for root of tree

INDUCTIVE STEP:

Try to prove P is true for an arbitrary node x.

INDUCTIVE HYPOTHESIS:
Assume P is true for all ancestors of x

∴ Conclude P is true for all nodes.