CS340: Discrete Structures **Homework #7**

Name _____

Due: Beginning of Class Monday June 7, 2010.

Hand in hard copy. Staple all pages.

1. Prove that the following wff is a tautology by using the CP rule. Do not use any T's (theorems).

$$(A \vee B \rightarrow C) \wedge (C \vee D \rightarrow E \wedge F) \rightarrow (A \rightarrow F)$$

2. Prove that the following wff is a tautology by using the CP rule. Do not use any T's.

$$(\neg A \vee \neg B) \wedge (B \vee C) \wedge (C \rightarrow D) \rightarrow (A \rightarrow D)$$

3. Prove that the following wff is a tautology by using the IP rule. Use T only for false.

$$(A \vee B) \wedge (A \rightarrow B) \rightarrow B$$

4. Describe a model for each of the following wffs.

a. $\exists x\, p(x) \wedge \exists x \neg p(x)$

b. $\forall x\, \exists y\, (p(x, y) \wedge \neg p(y, x))$

5. Describe a countermodel for each of the following wffs.

a. $\exists x\, p(x) \rightarrow \forall x\ p(x)$

b. $\exists x\, p(x) \wedge \exists x\ q(x) \rightarrow \exists x\ (p(x) \wedge q(x))$

c. $\forall y\, \exists x\ p(x, y) \rightarrow \exists x\, \forall y\, p(x, y)$

6. Let $C(x)$ mean x is a child, $V(x)$ mean x is a vegetable, and $L(x, y)$ mean x likes y. Find a wff to formalize each of the following English sentences.

a. *"Every child hates some vegetable."*

b. *"Some child hates all vegetables."*

c. *"Only adults like vegetables."*

7. Prove the correctness of the following wff where x and y are integers.

$\{x > y + 1\}\ x := x - 1;\ y := y + 1\ \{x \geq y\}$

8. Prove the correctness of the following wff, where x and y take integer values.

$\{\exists x\ (y = 2x)\}\ y := y - 5\ \{\exists x\ (y = 2x + 1)\}$