# CS 340: Discrete Structures for Engineers

**Instructor:** Prof. Harry Porter harry@cs.pdx.edu

**Office:** FAB 115-06

**Hours:** Mon 3-4, Wed 3-4, or by appointment

**Website:** web.cecs.pdx.edu/~harry/discrete

**Class Mailing List:** PorterClassList2 (mailman)

**Important Dates:**

Exam #1 –

Exam #2 –

Final Exam - Thursday, June 10, 12:30PM

Holiday - Monday, May 31

**Grading:**

10% Homeworks

10% In-class quizzes, class participation

50% Midterm exams

30% Final Exam

**Study Habits:**

Study the glossary of symbols and definitions;

Get to know the "language".

Proficiency will come from doing problems.

Go beyond the assigned work.

Look at problems early so your subconscious has plenty of time to play with them.

Review daily

Read ahead, before each lecture.

Don't expect immediate success.

Anything worthwhile takes time and effort.

**Problem solving proficiency will be key to your success!**

# Slide Credits:

# *Chris Brooks*

## Section 1.1: What is a Proof?

A proof is a demonstration that some statement is true.
(There are other non-mathematical definitions.)

A statement is "true" iff (if-and-only-if) we have a rigorous proof.

We normally express proofs using English sentences mixed with symbols.
We take a statement to be either true or false.

If A and B are statements,

- not A
  - **negation** – opposite in truth value from A
- A and B
  - **conjunction** – true exactly when both A and B are true
- A or B
  - **disjunction** – true except when both A and B are false
- if A then B
  - **conditional** statement – A is the **hypothesis**, B is the **conclusion**
    - Its **contrapositive** is "if not B then not A"
    - Its **converse** is "if B then A"

## Truth Tables

| A | B | not A | A and B | A or B | if A then B | if not B then not A |
|---|---|---|---|---|---|---|
| T | T | F | T | T | T | T |
| T | F | F | F | T | F | F |
| F | T | T | F | T | T | T |
| F | F | T | F | F | T | T |

Statements with the same truth table values are **equivalent**.

This table shows that a conditional and its contrapositive are equivalent.

We'll demonstrate proofs using numbers.

**Some definitions:**

**integers** {…, -2, -1, 0, 1, 2, …}
**odd** integers {…, -3, -1, 1, 3, …}
They have the form 2k+1 for some integer k.
**even** integers {…, -4, -2, 0, 2, 4, …}
They have the form 2k for some integer k.
m | n "m **divides** n" if m ≠ 0 and n = km for some integer k
p is **prime** if p>1 and its only divisors are 1 and p

**Characteristics of a good proof:**

- It is clear and correct.
- It has a nice structure, like a good program.
  Broken up into separate parts that define and prove intermediate steps. Pieces are decomposable, independent.
- Easy to understand, follow, verify.
  Like a good scientific experiment: easy to replicate.

## Proof Approach #1: Exhaustive Checking

Some statements can be proven by checking all possible cases
Must be a finite number.

*Example:* There is a prime number between 200 and 220.
*Proof:* Check exhaustively. Find that 211 is prime. QED

*Example:* Each of the numbers 288, 198, and 387 is divisible by 9.
*Proof:* Check that 9 divides each number. QED

## Proof Approach #2: Conditional Proof

Most statements we prove are conditionals.

if A then B

Start by assuming the hypothesis is true.

Then try to find a statement that follows from the hypothesis and/or known facts.

Continue deriving new statements until we reach the conclusion.

*Example:* If x is odd and y is even then x-y is odd.

*Proof:* Assume x is odd and y is even. Then $x=2k+1$ and $y=2m$ for some integers k and m. So we have

$$x-y = (2k+1) - (2m) = 2(k-m) + 1$$

Since $k-m$ must be an integer, $2(k-m)+1$ must be odd. QED

*Example:* If x is odd then $x^2$ is odd.

*Proof:* Assume x is odd. Then $x=2k+1$ for some integer k. So we have

$$x2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2+2k) + 1$$

Since $2k^2+2k$ must be an integer, $2(2k^2+2k)+1$ must be odd. QED

*Example:* If x is even then $x^2$ is even.
*Proof:*
**In-class quiz**

*Example:* If $x^2$ is odd, then x is odd.
*Proof:* The contrapositive of this statement is:
If x is even, then $x^2$ is even
which was proven in the previous example. QED

*Example:* If $x^2$ is even, then x is even.
*Proof:*
**In-class quiz**

## If and Only If Proofs – iff

A statement of the form "A if and only if B" means

A iff B

"A implies B" and "B implies A"

Approach #1: Prove "A implies B". Then prove "B implies A".

Approach #2: Create a chain of statements.

A iff $X_1$ iff $X_2$ iff … iff $X_n$ iff B

*Theorem:* x is even if and only if $x^2$-2x+1 is odd.

*Proof:*

| | | |
|---|---|---|
| | x is even | |
| iff | x = 2k for some integer k | (definition) |
| iff | x-1 = 2k-1 for some integer k | (algebra) |
| iff | x-1 = 2(k-1)+1 for some integer k | (algebra) |
| iff | x-1 is odd | (definition) |
| iff | $(x-1)^2$ is odd | (previous example) |
| iff | $x^2$-2x+1 is odd | (algebra) |

QED

A false statement is called a contradiction.
For example, “S and not S” is a contradiction for and statement S.

**Proof By Contradiction**
A truth table shows that
    “if A then B”
is equivalent to
    “A and not B implies false”
To prove “if A then B”, start by assuming “A” and assuming “not B”.
Then argue toward a false statement… the contradiction.
If, from “A” and “not B” you can derive a statement that is false,
    you’ve found a “proof by contradiction”. “B” must really be true.

*Theorem:* If $x^2$ is odd then x is odd.
*Proof:* Assume that $x^2$ is odd and x is even.
Then $x=2k$ for some integer k.
So we have
$$x^2 = (2k)^2 = 4k^2 = 2(2k^2)$$
Since $2k^2$ is an integer, $2(2k^2)$ must be even.
So $x^2$ is even.
From the assumption we have
$x^2$ is odd *and* $x^2$ is even
This is a contradiction, proving the theorem is true! QED

*Theorem:* if $2 \mid 5n$ then n is even.
*Proof:* Assume that $2 \mid 5n$ and n is odd.
Since $2 \mid 5n$, we have $5n = 2d$ for some integer d.
Since n is odd, we have $n = 2k+1$ for some integer k.
Then we have
$$2d = 5n = 5(2k+1) = 10k+5$$
So $2d = 10k+5$.
Rewriting, we get $5 = 2d-10k = 2(d-5k)$
Since $d-5k$ is an integer, we see that 5 is apparently a even number.
False! Contradiction! So the theorem is proven. QED