CS 311 Homework 2

October 7, 2013

1. (1.14 from Sipser)
   (a) Show that if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.
   (b) Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.
2. (1.8 from Sipser) Use the construction in the proof of Theorem 1.45 to give the state diagram of NFAs recognizing the union of the following languages
   (a) $\{w|w \text{ begins with a 1 and ends with an 0}\}$ and $\{w|w \text{ contains at least three } 1s\}$
   (b) $\{w|w \text{ contains the substring } 1010\}$ and
   $\{w|w \text{ has length at least three and the third character is } 0\}$
3. (1.9 from Sipser) Use the construction in the proof of Theorem 1.47 to give the state diagrams of NFAs recognizing the concatenation of the following languages
   (a) $\{w| \text{ the length of } w \text{ is at least } 5\}$ and $\{w| \text{ every odd position of } w \text{ is a } 1\}$
   (b) $\{w|w \text{ contains at least three } 1s\}$ and the empty set
4. (1.10 from Sipser) Use the construction in the proof of Theorem 1.49 to give the state diagram of NFAs recognizing the star of the following languages
   (a) $\{w|w \text{ contains at least three } 1s\}$
   (b) the empty language
5. (1.31 from Sipser, problem 5 from the last homework) For any string $w = w_1 w_2 \dots w_n$, the reverse of w, written as w^R is the string w in reverse order $w_n \dots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, then so is A^R.

6. (1.32 from Sipser) Let

$$\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \ldots \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$$

i.e. Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let B be the language

$$B = \{w \in \Sigma_3^* | \text{the bottom row of } w \text{ is the sum of the top two rows}\}$$

Show that B is regular. (Hint: Working with B^R is easier. You may assume the result claimed in the previous problem.)