CS 311: Computational Structures

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6 PDA to CFG Example

Consider the PDA:

$$
\begin{array}{lcll}
\delta(0,\epsilon,\epsilon) & = & \{(1,\$)\} & +\$ \quad 1 \\
\delta(1,a,\epsilon) & = & \{(1,\#)\} & +\# \quad 2 \\
\delta(1,b,\#) & = & \{(2,\epsilon)\} & -\# \quad 3 \\
\delta(2,b,\#) & = & \{(2,\epsilon)\} & -\# \quad 4 \\
\delta(2,\epsilon,\$) & = & \{(3,\epsilon)\} & -\$ \quad 5
\end{array}
$$

In this summary I have indicated if a rule is a "push" of t ($+t$) or a "pop" of t ($-t$). I have also numbered each line in the definition of δ for reference.

Recall that the construction introduces rules of the form:

$$A_{pq} \to aA_{rs}b$$

when there is a stack symbol t such that:

$$
\begin{array}{lcl}
(r,t) & \in & \delta(p,a,\epsilon) \\
(q,\epsilon) & \in & \delta(s,b,t)
\end{array}
$$

Note that this is exactly when the transition from p to r is labeled $+t$ and the transition from s to q is labeled $-t$.

Applying this rule to all of δ yields 3 instances. They are:

$$
\begin{array}{lcllr}
A_{03} & \to & A_{12} & +-\$ & 1,5 \\
A_{12} & \to & aA_{11}b & +-\# & 2,3 \\
 & | & aA_{12}b & +-\# & 2,4
\end{array}
$$

Here I have annotated each rule with what symbol is being pushed and poped ($+-t$) and which lines in the definition of δ are used in the construction.

The grammar is completed by using one instance of the construction that introduces null productions:

$$A_{11} \to \epsilon$$

It is, of course, safe to add all other null productions, but no other null productions contribute to the generation of any strings in the language.