

# CS311 – Computational Structures – HW4

Thursday, April 21, 2011  
due in class Thursday, April 28, 2011

Answer each question below. Write your answers neatly on paper. Be sure your name is on the paper, and the paper is clearly identified as Homework 4.

1. Problem 1, Section 11.4, page 755. (Parts a, e, & i) RegExp  $\rightarrow$  RegGrammar. 5 points each. In this problem its sufficient to just write down the grammar.
2. Problem 4, Section 11.4, page 755. Left Biased RegGrammars. 5 points each. Write down the grammars. Explain in a sentence or two why the two formulations of Regular Grammars are equivalent.
3. Problem 6, Section 11.4, page 756. RegGrammar to NFA 10 points. Use labels on the NFA that correspond to the grammar where possible. Show enough steps to demonstrate you know the algorithm.
4. Problem 7, Section 11.4, page 756. (Parts a, b, & c) Pumping Lemma. 10 points each. A pumping lemma argument is always a proof by contradiction. Clearly state what you are assuming to be true. Clearly state how that leads to a contradiction. Restate what the contradiction shows to be true.
5. We argued in class that if  $L_1$  and  $L_2$  are regular languages, then the intersection of  $L_1$  and  $L_2$  is a regular language. Demonstrate this by drawing two DFA's over the alphabet  $\{a,b,c\}$ , the first for strings with even lengths, and the second for strings with exactly one 'a'. Show that their intersection, the strings (over the alphabet  $\{a,b,c\}$ ) of even length with exactly one 'a', is regular by computing a DFA for that language. Build the DFA by using the product construction discussed in class on Tuesday, April 19. Show enough steps to demonstrate you are using the algorithm. Label the states of the new DFA by pairs of labels from  $L_1$  and  $L_2$ . Draw the resulting DFA. 20 points.
6. Prove that for all natural numbers  $n$ , when  $n \geq 5$ , it holds that  $2^n > n^2$ .
  - What is the induction variable? (1 points)
  - What is the formula as a function of the induction variable. (1 points)
  - Use the mathematical induction to formulate the structure of the proof. There something that makes this proof a bit different from other proofs we've seen over natural numbers. What is it? (1 points) Write down the cases of the proof. (1 points) Which cases have induction hypotheses?
  - Carry out the steps of the proof, label each step with the properties you use. (3 points)
  - Write down any facts about arithmetic that you use in your proof. (3 points)