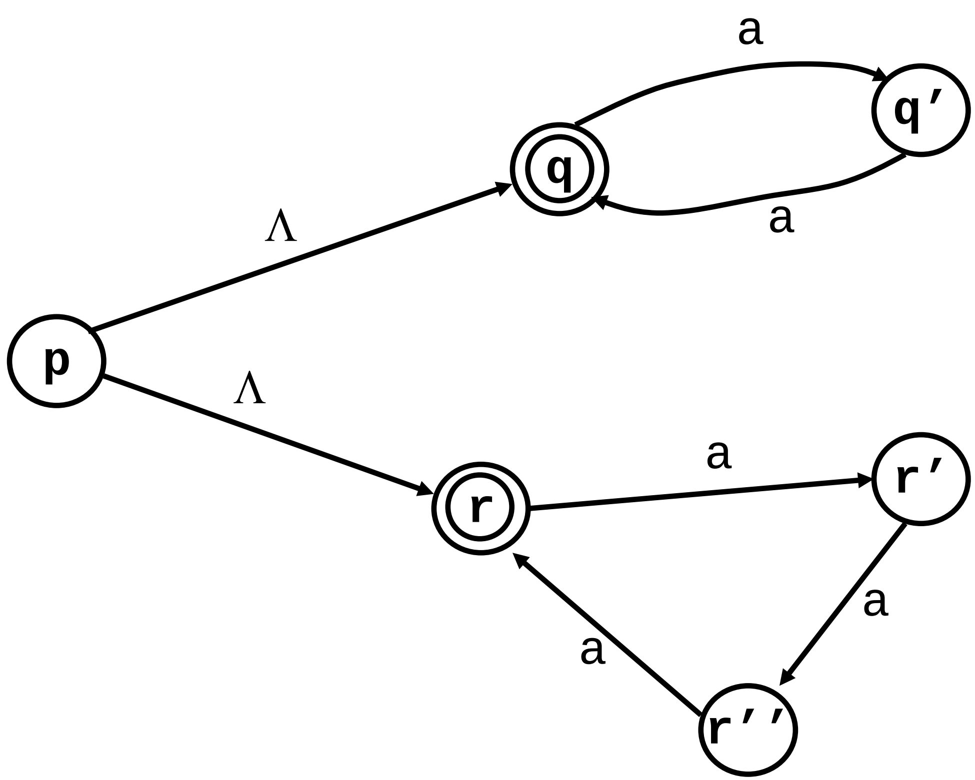
NFA’s with Λ–Transitions

- We extend the class of NFAs by allowing instantaneous transitions:
  1. The automaton may be allowed to change its state without reading the input symbol.
  2. In diagrams, such transitions are depicted by labeling the appropriate arcs with Λ.
  3. Note that this does not mean that Λ has become an input symbol. On the contrary, we assume that *the symbol* Λ *does not belong to any alphabet*.

example

- { a^n | n is even or divisible by 3 }



Definition

- **A** ε-NFA is a quintuple **A**$=($**Q**$,\Sigma,$**s**$,$**F**$,\delta),$ where

  - **Q** is a set of *states*
  - Σ is the alphabet of *input symbols*
  - **s** is an element of **Q** --- the *initial state*
  - **F** is a subset of **Q** ---the set of *final states*
  - δ: **Q** $\times$ $(\Sigma \cup \Lambda) \longrightarrow$ **Q** is the *transition function*
- Note Λ is never a member of Σ

ε-NFA

- ε-NFAs add a convenient feature but (in a sense) they bring us nothing new: they do not extend the class of representable languages.

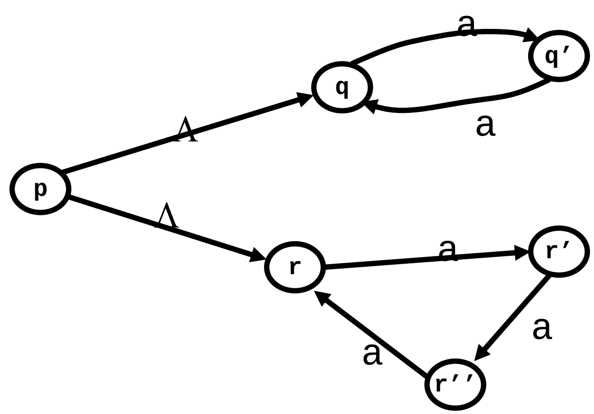
- **Theorem**. Every language accepted by an ε-NFA is also accepted by some DFA.

- The proof requires a modification of the subset construction. To describe it, we need the notion of Λ-closure.

- Λ-transitions are a convenient feature: try to design an NFA for the even or divisible by 3 language that does not use them!

Λ-Closure

- Λ-closure of a state
- The *Λ-closure* of the state q, denoted ECLOSE(q), is the set that contains q, together with all states that can be reached starting at q *by following only Λ-transitions*.



- In the above example:
- ECLOSE(p) ={p,q,r}
- ECLOSE(x)={x} for any of the remaining five states, x.

Elimination of Λ-Transitions

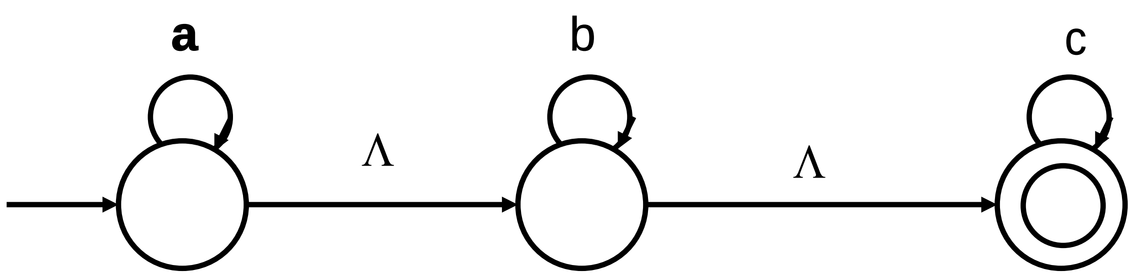
- Given an ε-NFA N, this construction produces an NFA N' such that L(N')=L(N).
- Then we can apply the subset construction to N and obtain a DFA, D, such that L(D)=L(N')=L(N). This would prove the Theorem (page 6) above.

- The construction of N' begins with N as input, and takes 3 steps:

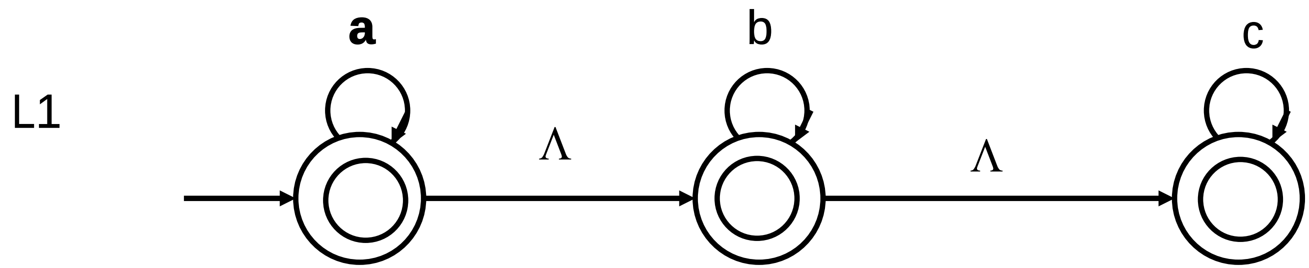
1. Make p an accepting state of N' iff ECLOSE(p) contains an accepting state of N.
2. Add an arc from p to q labeled a iff there is an arc labeled a in N from some state in ECLOSE(p) to q.
3. Delete all arcs labeled Λ.

Illustration

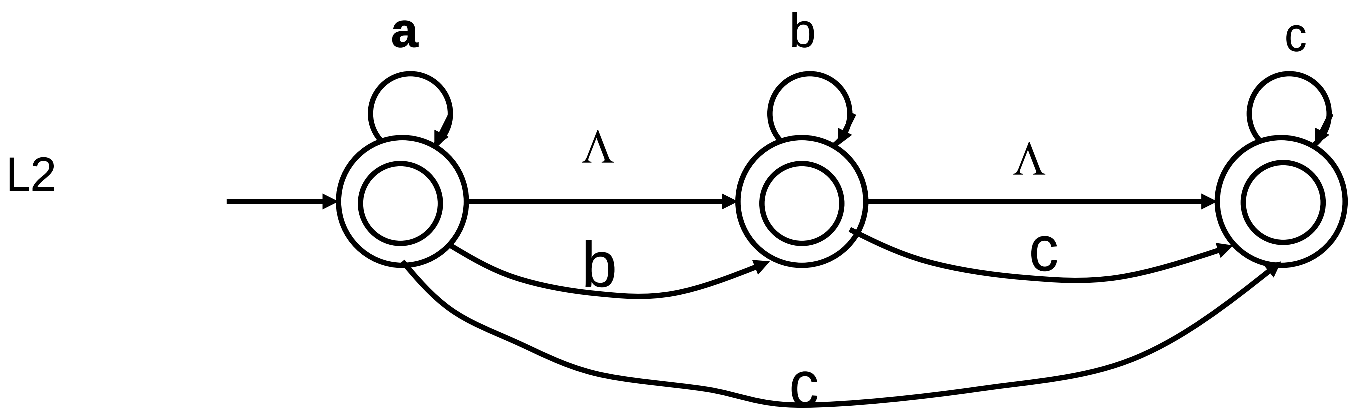
- We illustrate the procedure on the following ε-NFA N, accepting the strings over *{a,b,c}* of the form $a^i b^j c^k$ ($i,j,k \geq 0$)



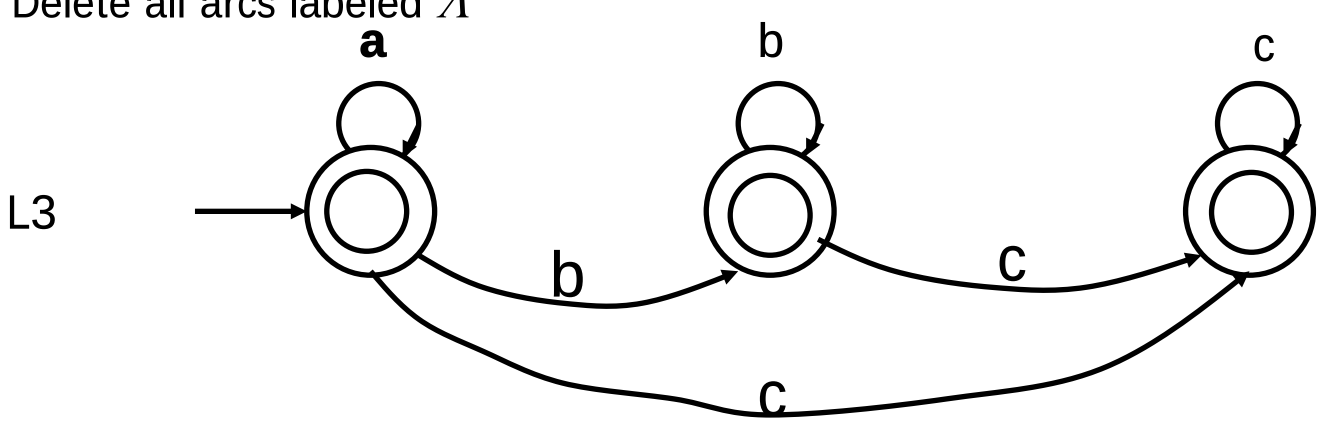
1) Make p an accepting state iff ECLOSE(p) contains an accepting state of N



2) Add an arc from p to q labeled a iff there is an arc labeled a from some state in ECLOSE(p) to q



3) Delete all arcs labeled Λ



Why does it work?

- The language accepted by the automaton is being preserved during the three steps of the construction: $L(N)=L(N_1)=L(N_2)=L(N_3)$

- Each step here requires a proof. A Good exercise for you to do!

Automata and Languages

- We have seen that the three types of automata we've considered all define the same class of languages:

- $\{L(A) \mid A \text{ is a DFA}\}$
- $= \{L(A) \mid A \text{ is an NFA}\}$
- $= \{L(A) \mid A \text{ is an } \varepsilon\text{-NFA}\}$

Remarkable facts

- A remarkable fact is that this class of languages is closed under boolean operations (union, intersection, complement) and Kleene star.
- i.e. if A, B are DFA's
- Then so are A^* $A \cup B$ $A \cap B$ and $\underline{A}$
- The converse is just as amazing: every language that can be obtained starting with finite subsets of an alphabet by applying these operations is a language of some DFA.
- It is important to understand these facts.