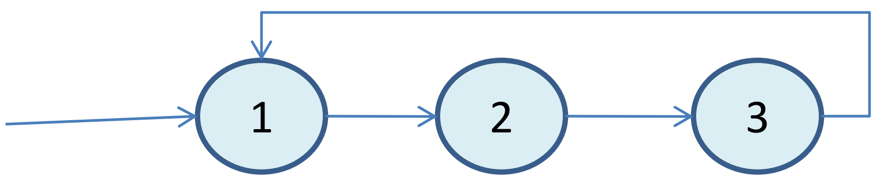
Putting Laziness to Work

Why use laziness

- Laziness has lots of interesting uses
  - Build cyclic structures. Finite representations of infinite data.
  - Do less work, compute only those values demanded by the final result.
  - Build infinite intermediate data structures and actually materialize only those parts of the structure of interest.
    - Search based solutions using enumerate then test .
  - Memoize or remember past results so that they don't need to be recomputed

Cyclic structures

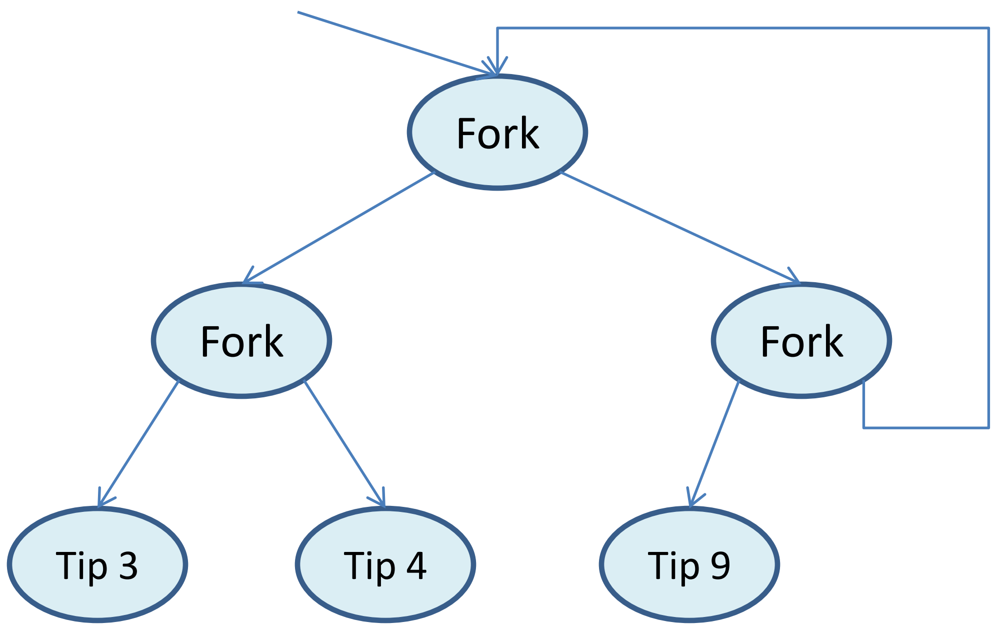
- cycles:: [Int]
- cycles = 1 : 2 : 3 : cycles



Cyclic Trees

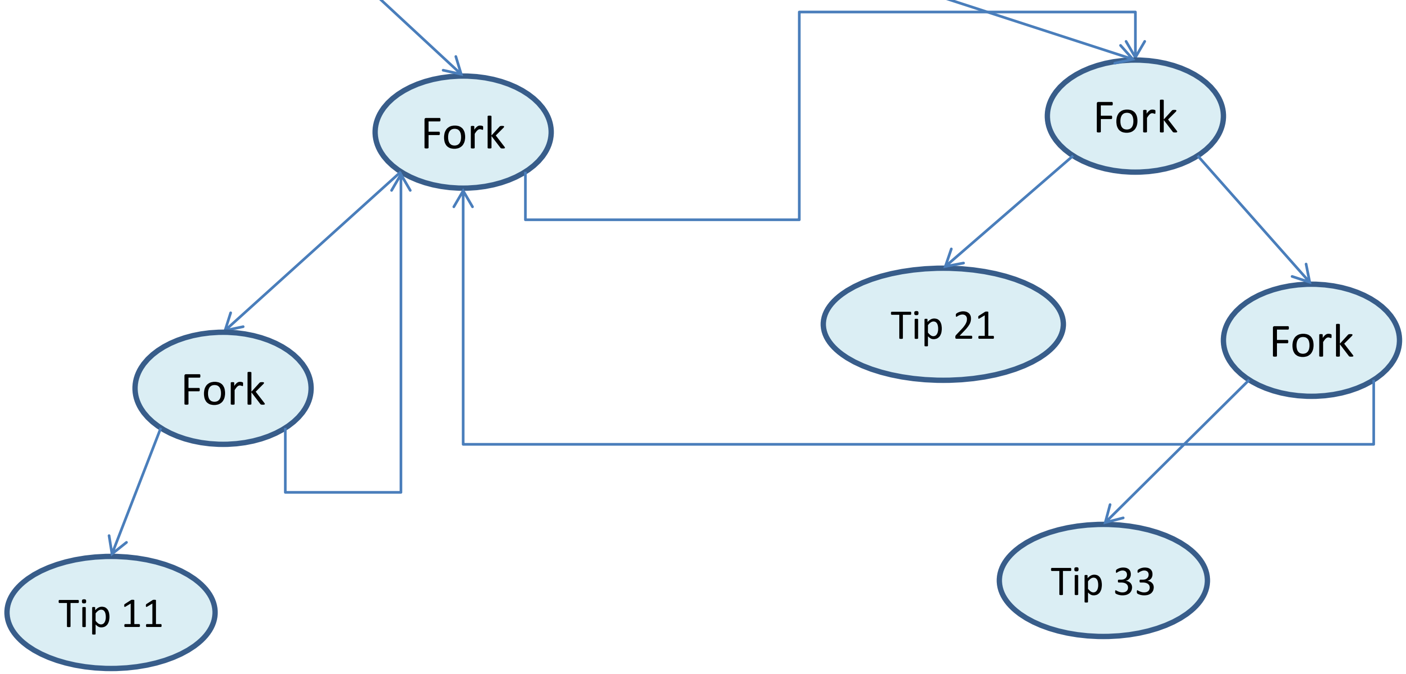
- data Tree a = Tip a | Fork (Tree a) (Tree a)

- t2 = Fork (Fork (Tip 3) (Tip 4)) (Fork (Tip 9) t2)



Mutually Cyclic

```
(t3,t4) = ( Fork (Fork (Tip 11) t3) t4
          , Fork (Tip 21) (Fork (Tip 33) t3)
          )
```



Prime numbers and infinite lists

```
primes :: [Integer]
primes = sieve [2..]
  where sieve (p:xs) =
           p : sieve [x | x<-xs
                        , x `mod` p /= 0]
```

Dynamic Programming

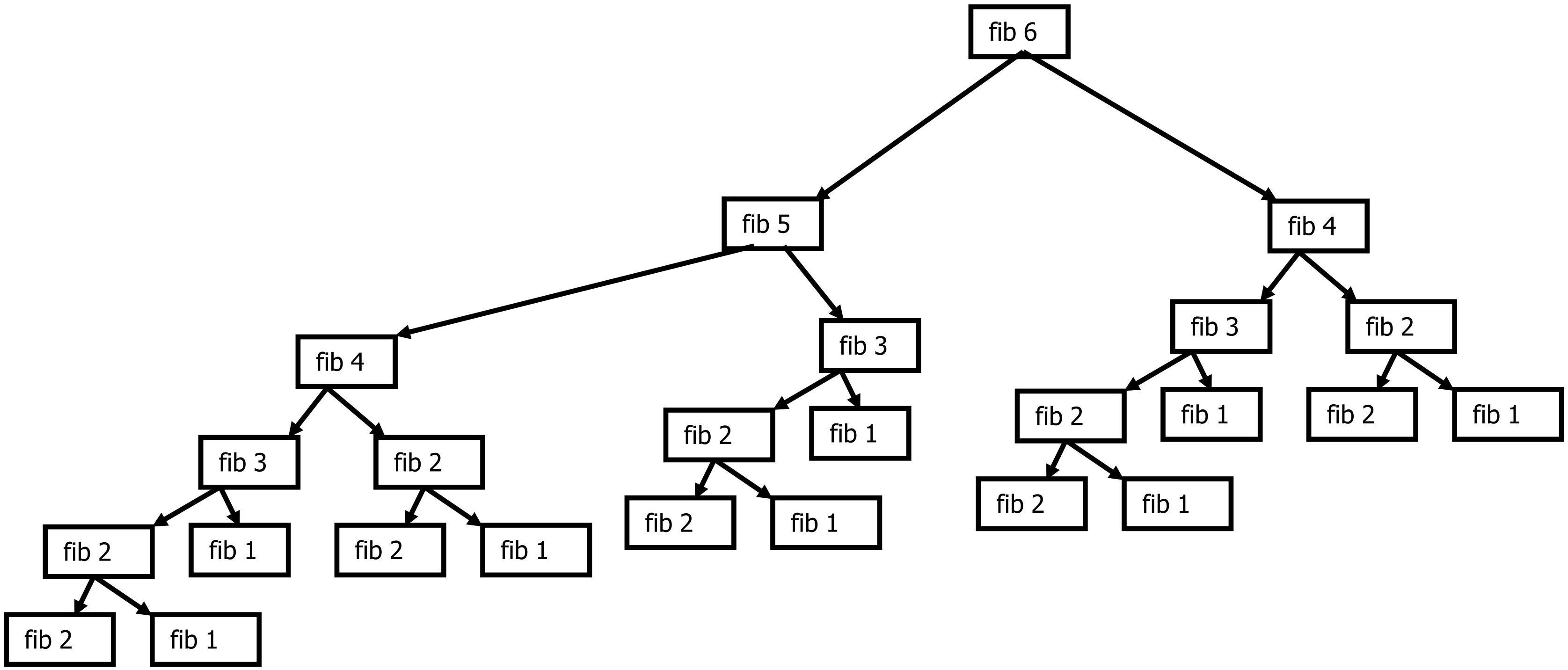
- Consider the function

```
fib :: Integer -> Integer
fib 0 = 1
fib 1 = 1
fib n = fib (n-1) + fib (n-2)

LazyDemos> :set +s
LazyDemos> fib 30
1346269
(48072847 reductions, 78644372 cells, 1 garbage
  collection)
```

- takes about 9 seconds on my machine!

Why does it take so long?

- Consider `(fib 6)`



What if we could remember past results?

- Strategy
  - Create a data structure
  - Store the result for every (fib n) only if (fib n) is demanded.
  - If it is ever demanded again return the result in the data structure rather than re-compute it
- Laziness is crucial
- Constant time access is also crucial
  - Use of functional arrays

Lazy Arrays

```
import Data.Array
table = array (1,5)
           [(1,'a'),(2,'b'),(3,'c'),(5,'e'),(4,'d')]
```

- The array is created once
- Any size array can be created
- Slots cannot be over written
- Slots are initialized by the list
- Constant access time to value stored in every slot

Taming the duplication

```
fib2 :: Integer -> Integer
fib2 z = f z
  where table = array (0,z) [ (i, f i) | i <- range (0,z) ]
        f 0 = 1
        f 1 = 1
        f n = (table ! (n-1)) + (table ! (n-2))
```

```
LazyDemos> fib2 30
1346269
(4055 reductions, 5602 cells)
```

Result is instantaneous on my machine

Can we abstract over this pattern?

- Can we write a memo function that memoizes another function.
- Allocates an array
- Initializes the array with calls to the function
- But, We need a way to intercept recursive calls

A fixpoint operator does the trick

- `fix f = f (fix f)`

- `g fib 0 = 1`
- `g fib 1 = 1`
- `g fib n = fib (n-1) + fib (n-2)`

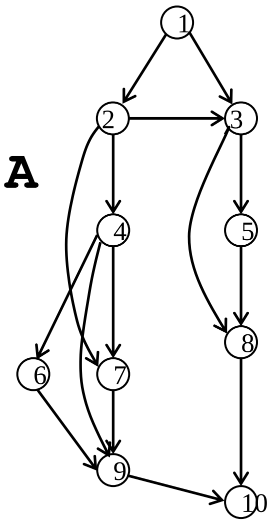
- `fib1 = fix g`

Generalizing

```
memo :: Ix a => (a,a) -> ((a -> b) -> a -> b) -> a -> b
memo bounds g = f
  where arrayF = array bounds
                      [ (n, g f n) | n <- range bounds ]
        f x = arrayF ! x


fib3 n = memo (0,n) g n



fact = memo (0,100) g
   where g fact n =
           if n==0 then 1 else n * fact (n-1)
```

Representing Graphs

```
import ST
import qualified Data.Array as A
type Vertex  = Int

-- Representing graphs:

type Table a = A.Array Vertex a
type Graph   = Table [Vertex]
               -- Array Int [Int]
```



1	[2,3]
2	[7,4]
3	[5]
4	[6,9,7]
5	[8]
6	[9]
7	[9]
8	[10]
9	[10]
10	[]

Index for each node

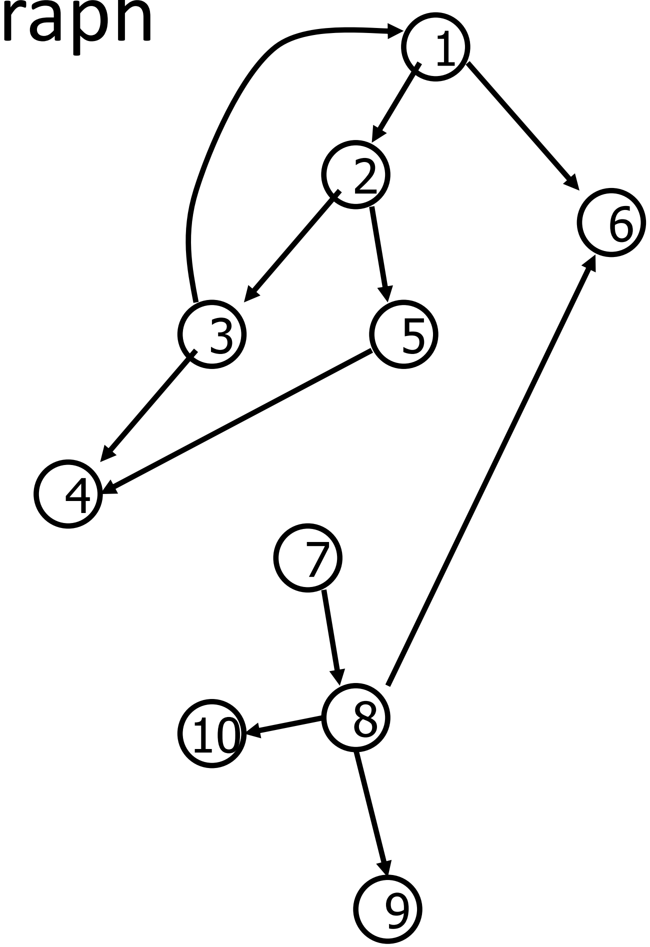
Edges (out of) that index

Functions on graphs

```
type Vertex = Int
type Edge = (Vertex,Vertex)

vertices :: Graph -> [Vertex]

indices :: Graph -> [Int]

edges :: Graph -> [Edge]
```

Building Graphs

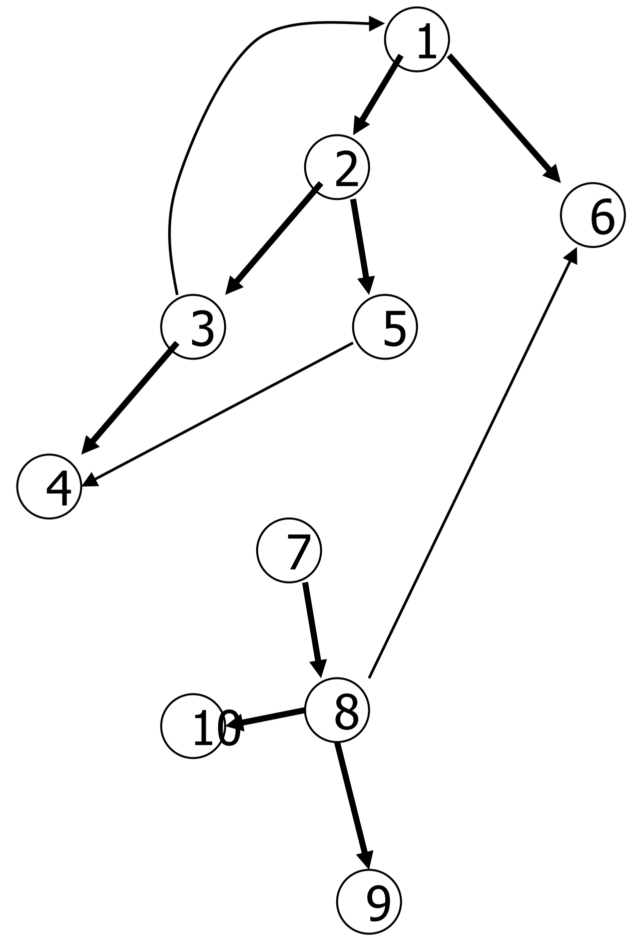
```
buildG :: Bounds -> [Edge] -> Graph

graph = buildG (1,10)
        [ (1, 2),  (1, 6),  (2, 3),
          (2, 5),  (3, 1),  (3, 4),
          (5, 4),  (7, 8),  (7, 10),
          (8, 6),  (8, 9),  (8, 10) ]
```



DFS and Forests

```
data Tree a   = Node a (Forest a)
type Forest a = [Tree a]


nodesTree (Node a f) ans =
   nodesForest f (a:ans)

nodesForest [] ans = ans
nodesForest (t : f) ans =
   nodesTree t (nodesForest f ans)
```

- Note how any tree can be spanned
- by a Forest. The Forest is not always
- unique.



DFS

- The DFS algorithm finds a spanning forest for a graph, from a set of roots.

An infinite cyclic tree

```
dfs :: Graph -> [Vertex] -> Forest Vertex

dfs          :: Graph -> [Vertex] -> Forest Vertex
dfs g vs      = prune (A.bounds g) (map (generate g) vs)

generate     :: Graph -> Vertex -> Tree Vertex
generate g v  = Node v (map (generate g) (g `aat` v))
```

Array indexing

Sets of nodes already visited

Mutable array

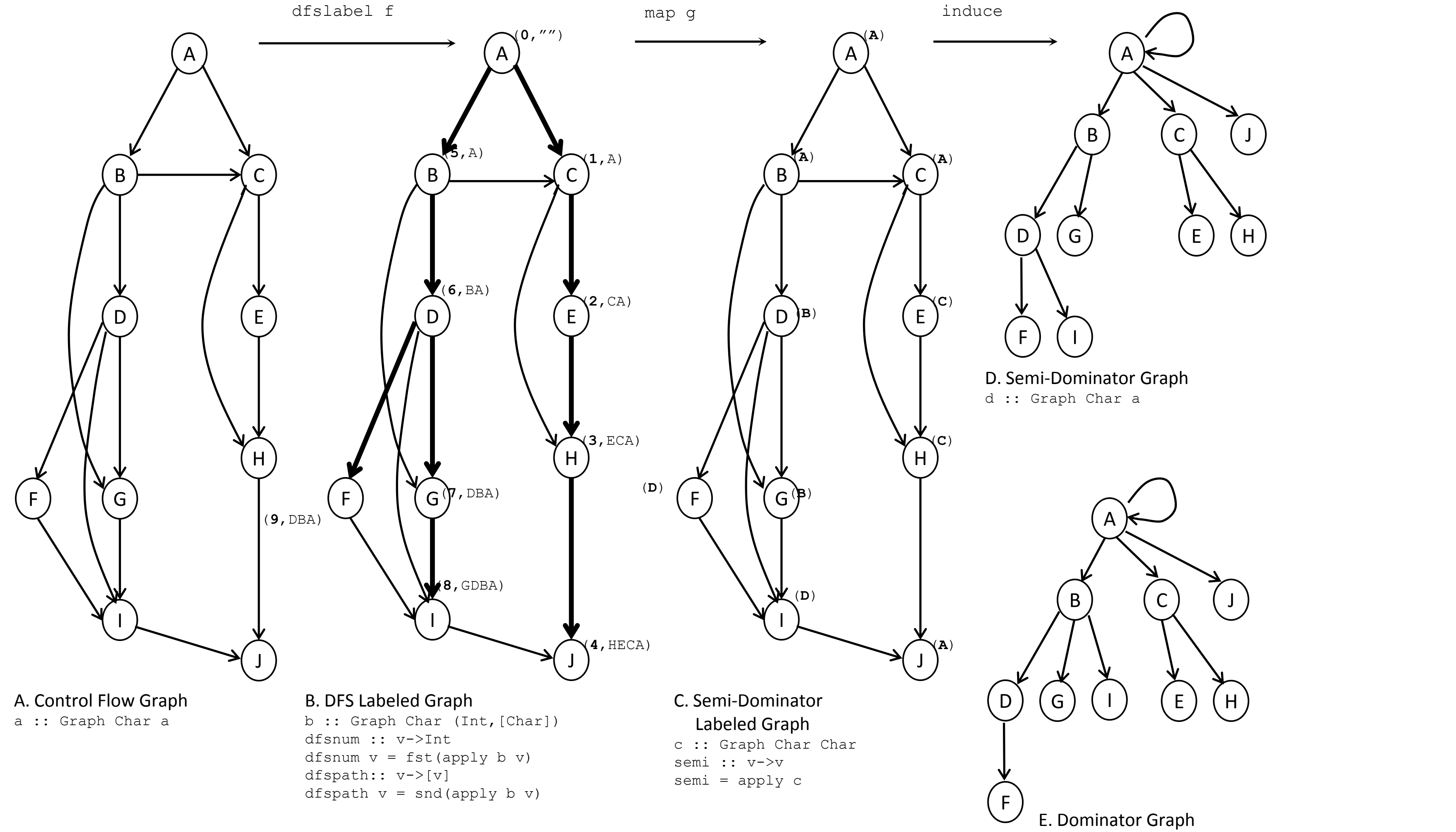
```
import qualified Data.Array.ST as B
type Set s     = B.STArray s Vertex Bool

mkEmpty        :: Bounds -> ST s (Set s)
mkEmpty bnds   = newSTArray bnds False

contains       :: Set s -> Vertex -> ST s Bool
contains m v   = readSTArray m v

include        :: Set s -> Vertex -> ST s ()
include m v    = writeSTArray m v True
```

Pruning already visited paths

```
prune :: Bounds -> Forest Vertex -> Forest Vertex
prune bnds ts =
    runST (do { m <- mkEmpty bnds; chop m ts })

chop :: Set s -> Forest Vertex -> ST s (Forest Vertex)
chop m []     = return []
chop m (Node v ts : us)
  do { visited <- contains m v
     ; if visited
          then chop m us
          else do { include m v
                  ; as <- chop m ts
                  ; bs <- chop m us
                  ; return(Node v as : bs)
                  }
     }
```

Topological Sort

```
postorder :: Tree a -> [a]
postorder (Node a ts) = postorderF ts ++ [a]

postorderF   :: Forest a -> [a]
postorderF ts = concat (map postorder ts)

postOrd      :: Graph -> [Vertex]
postOrd       = postorderF . Dff

dff          :: Graph -> Forest Vertex
dff g         = dfs g (vertices g)
```



A. Control Flow Graph

`a :: Graph Char a`

B. DFS Labeled Graph

```
b :: Graph Char (Int,[Char])
dfsnum :: v->Int
dfsnum v = fst(apply b v)
dfspath:: v->[v]
dfspath v = snd(apply b v)
```

C. Semi-Dominator Labeled Graph

```
c :: Graph Char Char
semi :: v->v
semi = apply c
```

D. Semi-Dominator Graph

`d :: Graph Char a`

E. Dominator Graph

`e :: Graph Char a`