Scholarship Skills 2006 Exercise 6 – Revise Mathematics

Apply what you've learned to rewrite this proof. Due at the beginning of class, Tuesday, January 31

The Largest Prime

Suppose there were a largest prime number p_i. Then consider the product $\prod_{j=0}^{p_i-1} p_i - j$. Then $\left(\prod_{j=0}^{p_i-1} p_i - j\right) + 1$ cannot be divided evenly by any of the numbers up to p_i, $2, 3, 4, \ldots, p_i$ because each of these divides the left factor evenly, but not the right factor, hence not their sum. (Recall that if a_1 divides a_2 and $a_2 = a_3 + a_4$ then if a_1 divides a_3, it will also divide a_4.) Since we are assuming p_i is the largest prime, $\left(\prod_{j=0}^{p_i-1} p_i - j\right) + 1$ can have no prime factors greater than p_i, hence $\left(\prod_{j=0}^{p_i-1} p_i - j\right) + 1$ is a prime, and it is greater than p_i, since $\prod_{j=0}^{p_i-1} p_i - j \geq p_i$. This contradicts the maximality of p_i. Hence the assumption that p_i is the largest prime must be false, and so there is no largest prime.